1. 1.5 A

2. (c) Ohm’s law states that the voltage $V$ is directly proportional to the current $I$, according to $V = IR$, where $R$ is the resistance. Thus, a plot of voltage versus current is a straight line that passes through the origin.

3. (e) Since both wires are made from the same material, the resistivity $\rho$ is the same for each. The resistance $R$ is given by $R = \rho \frac{L}{A}$ (Equation 20.3), where $L$ is the length and $A$ is the cross-sectional area of the wire. With twice the length and one-half the radius (one-fourth the cross-sectional area), the second wire has $\frac{L}{A} = \frac{2}{1/4} = 8$ times the resistance as the first wire.

4. 250 °C

5. (a) Power $P$ is the current $I$ times the voltage $V$ or $P = IV$ (Equation 20.6a). However, since Ohm’s law applies to a resistance $R$, the power is also $P = I^2R$ (Equation 20.6b) and $P = \frac{V^2}{R}$ (Equation 20.6c). Therefore, all three of the changes specified leave the power unchanged.

6. 27 W

7. 0.29 A

8. (d) According to Ohm’s law, the voltage across the resistance $R_1$ is $V_1 = IR_1$. The two resistors are connected in series, and their equivalent resistance is, therefore, $R_1 + R_2$. According to Ohm’s law, the current in the circuit is $I = \frac{V}{R_1 + R_2}$. Substituting this expression into the expression for $V_1$ gives $V_1 = \left(\frac{V}{R_1 + R_2}\right)R_1$. 
9. (b) The series connection has an equivalent resistance of \( R_s = R + R = 2R \). The parallel connection has an equivalent resistance that can be determined from \( \frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \).

Therefore, it follows that \( R_p = \frac{1}{2} R \). The ratio of these values is \( \frac{R_s}{R_p} = \frac{2R}{\frac{1}{2} R} = 4 \).

10. (e) Since the two resistors are connected in parallel across the battery terminals, the same voltage is applied to each. Thus, according to Ohm’s law, the current in each resistor is inversely proportional to the resistance, so that \( I_1 = \frac{V}{R_1} \) and \( I_2 = \frac{V}{R_2} \). Therefore, \( I_1 I_2 = \frac{V}{R_1} \cdot \frac{V}{R_2} = \frac{V^2}{R_1 R_2} \).

11. 0.019 A

12. (c) In arrangement B the two resistors in series have a combined resistance of \( 2R \). Each series combination is in parallel, so that the reciprocal of the equivalent resistance is \( \frac{1}{R_{\text{eq}, B}} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R} \), or \( R_{\text{eq}, B} = R \). Following the technique outlined in Section 20.8, we also find that \( R_{\text{eq}, C} = \frac{3}{4} R \) and \( R_{\text{eq}, A} = \frac{5}{3} R \).

13. (d) The internal resistance \( r \) of the battery and the resistance \( R \) are in series, so that the current from the battery can be calculated via Ohm’s law as \( I = \frac{V}{R + r} \). The voltage between the terminals is, then, \( \frac{1}{2} V = IR = \frac{VR}{R + r} \). This result can be solved to show that \( R = r \).

14. (b) Kirchhoff’s junction rule states that the sum of the magnitudes of the currents directed into a junction equals the sum of the magnitudes of the currents directed out of the junction. Here, this rule implies that \( I_1 + I_3 = I_2 \).

15. (d) Once the current in the resistor is drawn, the markings of the plus and minus sign are predetermined. This is because conventional current always flows from a higher toward a lower potential. Thus, since the current \( I_4 \) is directed from right to left, the right side of \( R_4 \) must be marked plus and the left side minus.

16. (b) Kirchhoff’s loop rule states that, around any closed loop, the sum of the potential drops equals the sum of the potential rises.

17. (a) The ammeter must be connected so that the current that flows through the resistor \( R_2 \) also flows through the ammeter. The voltmeter must be connected across the resistor \( R_2 \).
18. (e) The two capacitors in series have an equivalent capacitance $C_S$ that can be determined from 
\[
\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C},
\]
so that $C_S = \frac{1}{2}C$. This capacitance is in parallel with a capacitance $C$, so that the total equivalent capacitance is 
\[
C_{eq} = \frac{1}{2}C + C = \frac{3}{2}C.
\]

19. (c) The time constant is given by the product of the resistance and the capacitance. Therefore, when the resistance is reduced to one-third of its initial value, the capacitance must be tripled, in order that the time constant remains unchanged.

20. 1.8 s
1. **REASONING** The current $I$ is defined in Equation 20.1 as the amount of charge $\Delta q$ per unit of time $\Delta t$ that flows in a wire. Therefore, the amount of charge is the product of the current and the time interval. The number of electrons is equal to the charge that flows divided by the magnitude of the charge on an electron.

**SOLUTION**

a. The amount of charge that flows is

$$\Delta q = I \Delta t = (18 \text{ A})(2.0 \times 10^{-3} \text{ s}) = 3.6 \times 10^{-2} \text{ C}$$

b. The number of electrons $N$ is equal to the amount of charge divided by $e$, the magnitude of the charge on an electron.

$$N = \frac{\Delta q}{e} = \frac{3.6 \times 10^{-2} \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 2.3 \times 10^{17}$$

2. **REASONING** We are given the average current $I$ and its duration $\Delta t$. We will employ $I = \frac{\Delta q}{\Delta t}$ (Equation 20.1) to determine the amount $\Delta q$ of charge delivered to the ground by the lightning flash.

**SOLUTION** Solving $I = \frac{\Delta q}{\Delta t}$ (Equation 20.1) for $\Delta q$, we obtain

$$\Delta q = I\Delta t = (1.26 \times 10^3 \text{ A})(0.138 \text{ s}) = 174 \text{ C}$$

3. **SSM REASONING AND SOLUTION** First determine the total charge delivered to the battery using Equation 20.1:

$$\Delta q = I \Delta t = (6.0 \text{ A})(5.0 \text{ h}) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 1.1 \times 10^5 \text{ C}$$

To find the energy delivered to the battery, multiply this charge by the energy per unit charge (i.e., the voltage) to get

$$\text{Energy} = (\Delta q)V = (1.1 \times 10^5 \text{ C})(12 \text{ V}) = 1.3 \times 10^6 \text{ J}$$
4. **REASONING** Knowing the resistance $R$ (14 $\Omega$) of the heating element and the voltage $V$ being applied to it (120 V), we can use $V = IR$ (Ohm’s law, Equation 20.2) to determine the current $I$.

**SOLUTION** Using Ohm’s law, we have

$$V = IR \quad \text{or} \quad I = \frac{V}{R} = \frac{120 \text{ V}}{14 \Omega} = 8.6 \text{ A}$$

5. **REASONING**
   a. According to Ohm’s law, the current is equal to the voltage between the cell walls divided by the resistance.

   b. The number of Na$^+$ ions that flow through the cell wall is the total charge that flows divided by the charge of each ion. The total charge is equal to the current multiplied by the time.

**SOLUTION**

a. The current is

$$I = \frac{V}{R} = \frac{75 \times 10^{-3} \text{ V}}{5.0 \times 10^9 \Omega} = 1.5 \times 10^{-11} \text{ A}$$

b. The number of Na$^+$ ions is the total charge $\Delta q$ that flows divided by the charge $+e$ on each ion, or $\Delta q/e$. The charge is the product of the current $I$ and the time $\Delta t$, according to Equation 20.1, so that

$$\text{Number of Na}^+ \text{ ions} = \frac{\Delta q}{e} = \frac{I \Delta t}{e} = \frac{(1.5 \times 10^{-11} \text{ A})(0.50 \text{ s})}{1.60 \times 10^{-19} \text{ C}} = 4.7 \times 10^7$$

6. **REASONING AND SOLUTION**
   a. The total charge that can be delivered is

$$\Delta q = (220 \text{ A} \cdot \text{h}) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 7.9 \times 10^5 \text{ C}$$

b. The maximum current is

$$I = \frac{220 \text{ A} \cdot \text{h}}{(38 \text{ min}) \left( \frac{1 \text{ h}}{60 \text{ min}} \right)} = 350 \text{ A}$$

7. **REASONING** According to Ohm’s law, the resistance is the voltage of the battery divided by the current that the battery delivers. The current is the charge divided by the time during which it flows, as stated in Equation 20.1. We know the time, but are not given the charge directly. However, we can determine the charge from the energy delivered to the resistor, because this energy comes from the battery, and the potential difference between the battery
terminals is the difference in electric potential energy per unit charge, according to Equation 19.4. Thus, a 9.0-V battery supplies 9.0 J of energy to each coulomb of charge passing through it. To calculate the charge, then, we need only divide the energy from the battery by the 9.0 V potential difference.

**SOLUTION** Ohm’s law indicates that the resistance \( R \) is the voltage \( V \) of the battery divided by the current \( I \), or \( R = V/I \). According to Equation 20.1, the current \( I \) is the amount of charge \( \Delta q \) divided by the time \( \Delta t \), or \( I = \Delta q/\Delta t \). Using these two equations, we have

\[
R = \frac{V}{I} = \frac{V}{\Delta q / \Delta t} = \frac{V \Delta t}{\Delta q}
\]

According to Equation 19.4, the potential difference \( \Delta V \) is the difference \( \Delta \text{(EPE)} \) in the electric potential energy divided by the charge \( \Delta q \), or \( \Delta V = \frac{\Delta \text{(EPE)}}{\Delta q} \). However, it is customary to denote the potential difference across a battery by \( V \), rather than \( \Delta V \), so \( V = \frac{\Delta \text{(EPE)}}{\Delta q} \). Solving this expression for the charge gives \( \Delta q = \frac{\Delta \text{(EPE)}}{V} \). Using this result in the expression for the resistance, we find that

\[
R = \frac{V \Delta t}{\Delta q} = \frac{V \Delta t}{\Delta \text{(EPE)}/V} = \frac{V^2 \Delta t}{\Delta \text{(EPE)}} = \frac{(9.0 \text{ V})^2 (6 \times 3600 \text{ s})}{1.1 \times 10^5 \text{ J}} = 16 \Omega
\]

8. **REASONING** Voltage is a measure of energy per unit charge (joules per coulomb). Therefore, when an amount \( \Delta q \) of charge passes through the toaster and there is a potential difference across the toaster equal to the voltage \( V \) of the outlet, the energy that the charge delivers to the toaster is given by

\[
\text{Energy} = V \Delta q \tag{1}
\]

The charge \( \Delta q \) that flows through the toaster in a time \( \Delta t \) depends upon the magnitude \( I \) of the electric current according to \( I = \frac{\Delta q}{\Delta t} \) (Equation 20.1). Therefore, we have that

\[
\Delta q = I \Delta t \tag{2}
\]

We will employ Ohm’s law, \( I = \frac{V}{R} \) (Equation 20.2), to calculate the current in the toaster in terms of the voltage \( V \) of the outlet and the resistance \( R \) of the toaster.

**SOLUTION** Substituting Equation (2) into Equation (1) yields

\[
\text{Energy} = V \Delta q = VI \Delta t \tag{3}
\]
Substituting $I = \frac{V}{R}$ (Equation 20.2) into Equation (3), we find that

$$\text{Energy} = VI \Delta t = V \left( \frac{V}{R} \right) \Delta t = \frac{V^2}{R} \Delta t = \frac{(120 \text{ V})^2}{14 \text{ } \Omega} (60.0 \text{ s}) = 6.2 \times 10^4 \text{ J}$$

9. **REASONING** The number $N$ of protons that strike the target is equal to the amount of electric charge $\Delta q$ striking the target divided by the charge $e$ of a proton, $N = \frac{\Delta q}{e}$. From Equation 20.1, the amount of charge is equal to the product of the current $I$ and the time $\Delta t$. We can combine these two relations to find the number of protons that strike the target in 15 seconds.

The heat $Q$ that must be supplied to change the temperature of the aluminum sample of mass $m$ by an amount $\Delta T$ is given by Equation 12.4 as $Q = cm\Delta T$, where $c$ is the specific heat capacity of aluminum. The heat is provided by the kinetic energy of the protons and is equal to the number of protons that strike the target times the kinetic energy per proton. Using this reasoning, we can find the change in temperature of the block for the 15 second-time interval.

**SOLUTION**

a. The number $N$ of protons that strike the target is

$$N = \frac{\Delta q}{e} = \frac{I \Delta t}{e} = \frac{(0.50 \times 10^{-6} \text{ A})(15 \text{ s})}{1.6 \times 10^{-19} \text{ C}} = 4.7 \times 10^{13}$$

b. The amount of heat $Q$ provided by the kinetic energy of the protons is

$$Q = (4.7 \times 10^{13} \text{ protons})(4.9 \times 10^{-12} \text{ J/proton}) = 230 \text{ J}$$

Since $Q = cm\Delta T$ and since Table 12.2 gives the specific heat of aluminum as $c = 9.00 \times 10^2 \text{ J/(kg}^\circ \text{C})$, the change in temperature of the block is

$$\Delta T = \frac{Q}{cm} = \frac{230 \text{ J}}{(9.00 \times 10^2 \frac{\text{ J}}{\text{kg} \cdot \circ \text{C}})(15 \times 10^{-3} \text{ kg})} = 17 \circ \text{C}$$

10. **REASONING**

a. The resistance $R$ of a piece of material is related to its length $L$ and cross-sectional area $A$ by Equation 20.3, $R = \rho \frac{L}{A}$, where $\rho$ is the resistivity of the material. In order to rank the resistances, we need to evaluate $L$ and $A$ for each configuration in terms of $L_0$, the unit of length.
Chapter 20   Problems

Resistance

<table>
<thead>
<tr>
<th>Resistance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R = \rho \frac{4L_0}{L_0 \times 2L_0} = \rho \frac{2}{L_0} )</td>
<td>1</td>
</tr>
<tr>
<td>( R = \rho \frac{L_0}{2L_0 \times 4L_0} = \rho \frac{1}{8L_0} )</td>
<td>3</td>
</tr>
<tr>
<td>( R = \rho \frac{2L_0}{L_0 \times 4L_0} = \rho \frac{1}{2L_0} )</td>
<td>2</td>
</tr>
</tbody>
</table>

Therefore, we expect that \( a \) has the largest resistance, followed by \( c \), and then by \( b \).

b. Equation 20.2 states that the current \( I \) is equal to the voltage \( V \) divided by the resistance, \( I = \frac{V}{R} \). Since the current is inversely proportional to the resistance, the largest current arises when the resistance is smallest, and vice versa. Thus, we expect that \( b \) has the largest current, followed by \( c \), and then by \( a \).

**SOLUTION**

a. The resistances can be found by using the results from the **REASONING**:

\[
\begin{align*}
  a & \quad R = \rho \left( \frac{2}{L_0} \right) = (1.50 \times 10^{-2} \, \Omega \cdot m) \left( \frac{2}{5.00 \times 10^{-2} \, m} \right) = 0.600 \, \Omega \\
  b & \quad R = \rho \left( \frac{1}{8L_0} \right) = (1.50 \times 10^{-2} \, \Omega \cdot m) \left( \frac{1}{8 \times 5.00 \times 10^{-2} \, m} \right) = 0.0375 \, \Omega \\
  c & \quad R = \rho \left( \frac{1}{2L_0} \right) = (1.50 \times 10^{-2} \, \Omega \cdot m) \left( \frac{1}{2 \times 5.00 \times 10^{-2} \, m} \right) = 0.150 \, \Omega
\end{align*}
\]

b. The current in each case is given by Equation 20.2, where the value of the resistance is obtained from part (a):

\[
\begin{align*}
  a & \quad I = \frac{V}{R} = \frac{3.00 \, V}{0.600 \, \Omega} = 5.00 \, A \\
  b & \quad I = \frac{V}{R} = \frac{3.00 \, V}{0.0375 \, \Omega} = 80.0 \, A \\
  c & \quad I = \frac{V}{R} = \frac{3.00 \, V}{0.150 \, \Omega} = 20.0 \, A
\end{align*}
\]

11. **REASONING**  The resistance \( R \) of a wire that has a length \( L \) and a cross-sectional area \( A \) is given by Equation 20.3 as \( R = \rho \frac{L}{A} \). Both wires have the same length and cross-sectional area. Only the resistivity \( \rho \) of the wire differs, and Table 20.1 gives the following values:
ρ_{Aluminum} = 2.82 \times 10^{-8} \, \Omega \cdot m \text{ and } \rho_{Copper} = 1.72 \times 10^{-8} \, \Omega \cdot m. \text{ Applying Equation 20.3 to both wires and dividing the two equations will allow us to eliminate the unknown length and cross-sectional area algebraically and solve for the resistance of the copper wire.}

**SOLUTION** Applying Equation 20.3 to both wires gives

\[ R_{Copper} = \rho_{Copper} \frac{L}{A} \text{ and } R_{Aluminum} = \rho_{Aluminum} \frac{L}{A} \]

Dividing these two equations, eliminating \( L \) and \( A \) algebraically, and solving the result for \( R_{Copper} \) give

\[ \frac{R_{Copper}}{R_{Aluminum}} = \frac{\rho_{Copper} L/A}{\rho_{Aluminum} L/A} = \frac{\rho_{Copper}}{\rho_{Aluminum}} \]

\[ R_{Copper} = R_{Aluminum} \left( \frac{\rho_{Copper}}{\rho_{Aluminum}} \right) = (0.20 \Omega) \left( \frac{1.72 \times 10^{-8} \, \Omega \cdot m}{2.82 \times 10^{-8} \, \Omega \cdot m} \right) = 0.12 \Omega \]

12. **REASONING** The materials in Table 20.1 are listed according to their resistivities, so we need to find the resistivity of this material. The resistivity depends on the resistance \( R \), the length \( L \), and the cross-sectional \( A \) of the wire (see Equation 20.3). We know the length of the wire, and the cross-sectional area can be found from the radius \( r \) since \( A = \pi r^2 \). The resistance depends on the voltage \( V \) and current \( I \) through the relation \( R = V / I \), (Equation 20.2).

**SOLUTION** The resistivity is

\[ \rho = \frac{RA}{L} = \frac{R(\pi r^2)}{L} \quad (20.3) \]

We also know that \( R = V / I \), so the resistivity becomes

\[ \rho = \frac{\left( \frac{V}{I} \right)(\pi r^2)}{L} = \frac{V \pi r^2}{IL} = \frac{(0.0320 \, V) \pi (1.03 \times 10^{-3} \, m)^2}{(1.35 \, A)(2.80 \, m)} = 2.82 \times 10^{-8} \, \Omega \cdot m \]

An inspection of Table 20.1 shows that the material that has this resistivity is aluminum.

13. **REASONING AND SOLUTION** Solving Equation 20.5 for \( \alpha \) yields

\[ \alpha = \frac{R}{R_0} \frac{1}{1 - \frac{43.7 \Omega}{38.0 \Omega}} = \frac{55 ^\circ C - 25 ^\circ C}{0.0050 (\circ C)^{-1}} = 0.0050 (\circ C)^{-1} \]
14. **REASONING** The resistance $R$ of the spooled wire decreases as its length $L$ decreases, according to $R = \rho \frac{L}{A}$ (Equation 20.3). The resistivity $\rho$ and cross-sectional area $A$ of the wire do not change. Because the same battery is used, the potential difference $V$ across the wire is the same in both cases. Therefore, $R = \frac{V}{I}$ (Equation 20.2) explains why the current $I$ increases as the resistance $R$ of the wire decreases.

**SOLUTION** Solving $R = \rho \frac{L}{A}$ (Equation 20.3) for $L_t$, we obtain $L_t = \frac{R_t A}{\rho}$. Thus, the initial length $L_0$ and final length $L_f$ of the wire are given by

$$L_0 = \frac{R_0 A}{\rho} \quad \text{and} \quad L_t = \frac{R_t A}{\rho} \quad \text{(1)}$$

where $R_0$ is the initial resistance, and $R_t$ the final resistance, of the wire. Taking the ratio of Equations (1) eliminates the unknown quantities $A$ and $\rho$, allowing us to solve for $L_f$ in terms of $L_0$ and the initial and final resistances:

$$\frac{L_f}{L_0} = \frac{R_f A}{R_0 A} \frac{\rho}{\rho} = \frac{R_f}{R_0} \quad \text{or} \quad L_f = \left( \frac{R_f}{R_0} \right) L_0 \quad \text{(2)}$$

From $R = \frac{V}{I}$ (Equation 20.2), the initial resistance $R_0$ and final resistance $R_t$ of the spooled wire are

$$R_0 = \frac{V}{I_0} \quad \text{and} \quad R_t = \frac{V}{I_t} \quad \text{(3)}$$

Substituting Equations (3) into Equation (2) yields

$$L_f = \left( \frac{\frac{V}{I_t}}{\frac{V}{I_0}} \right) L_0 = \left( \frac{I_0}{I_t} \right) L_0 = \left( \frac{2.4 \text{ A}}{3.1 \text{ A}} \right) (75 \text{ m}) = 58 \text{ m}$$

15. **SSM REASONING** The resistance of a metal wire of length $L$, cross-sectional area $A$ and resistivity $\rho$ is given by Equation 20.3: $R = \rho L / A$. Solving for $A$, we have $A = \rho L / R$. We can use this expression to find the ratio of the cross-sectional area of the aluminum wire to that of the copper wire.
**SOLUTION** Forming the ratio of the areas and using resistivity values from Table 20.1, we have

\[
\frac{A_{\text{aluminum}}}{A_{\text{copper}}} = \frac{\rho_{\text{aluminum}} L/R}{\rho_{\text{copper}} L/R} = \frac{2.82 \times 10^{-8} \ \Omega \cdot \text{m}}{1.72 \times 10^{-8} \ \Omega \cdot \text{m}} = 1.64
\]

16. **REASONING AND SOLUTION** Using Equation 20.3 and the resistivity of aluminum from Table 20.1, we find

\[
R = \frac{\rho L}{A} = \frac{(2.82 \times 10^{-8} \ \Omega \cdot \text{m})(10.0 \times 10^3 \ \text{m})}{4.9 \times 10^{-4} \ \text{m}^2} = 0.58 \ \Omega
\]

17. **REASONING** Assuming that the resistance is \(R\) at a temperature \(T\) and \(R_0\) at a temperature \(T_0\), we can write the percentage change \(p\) in resistance as

\[
p = \frac{R - R_0}{R_0} \times 100
\]

Equation 20.5, on the other hand, gives the resistance as a function of temperature as follows:

\[
R = R_0 \left[ 1 + \alpha (T - T_0) \right]
\]

where \(\alpha\) is the temperature coefficient of resistivity. Substituting this expression into the expression for the percentage change in resistance gives

\[
p = \frac{R - R_0}{R_0} \times 100 = \frac{R_0 + R_0 \alpha (T - T_0) - R_0}{R_0} \times 100 = \alpha (T - T_0) \times 100
\]

(1)

The change in temperature is unknown, but it is the same for both wires. Therefore, we will apply Equation (1) to each wire and divide the two expressions to eliminate the unknown change in temperature. From the result we will be able to calculate the percentage change in the resistance of the tungsten wire.

**SOLUTION** Applying Equation (1) to both wires gives

\[
p_{\text{Tungsten}} = \alpha_{\text{Tungsten}} (T - T_0) \times 100 \quad \text{and} \quad p_{\text{Gold}} = \alpha_{\text{Gold}} (T - T_0) \times 100
\]

Dividing these two expressions, eliminating \((T - T_0)\) algebraically, and solving for \(p_{\text{Tungsten}}\) give
18. **REASONING** The resistance $R$ of the wire depends on its length $L$, so we can use Equation 20.3 to express the length in terms of the resistance:

$$ L = \frac{RA}{\rho} $$

where $A$ is the cross-sectional area of the wire and $\rho$ is the resistivity of tungsten. The cross-sectional area can be expressed in terms of the radius $r$ of the wire since $A = \pi r^2$. The resistance depends on the voltage $V$ and current $I$ through the relation $R = V/I$ (Equation 20.2). Thus, the length of the wire can be expressed as

$$ L = \frac{RA}{\rho} = \frac{\left(\frac{V}{I}\right)(\pi r^2)}{\rho} $$

The resistivity $\rho$ at the temperature $T$ depends on the resistivity $\rho_0$ at the temperature $T_0$ through the relation $\rho = \rho_0 \left[1 + \alpha(T - T_0)\right]$ (Equation 20.4), where $\alpha$ is the temperature coefficient of resistivity. Substituting this expression for $\rho$ into the expression for the length of the wire, we have

$$ L = \frac{\left(\frac{V}{I}\right)(\pi r^2)}{\rho_0 \left[1 + \alpha(T - T_0)\right]} $$

**SOLUTION** The length of the wire is

$$ L = \frac{\left(\frac{V}{I}\right)(\pi r^2)}{\rho_0 \left[1 + \alpha(T - T_0)\right]} $$

$$ = \frac{\left(\frac{120 V}{1.5 A}\right)\pi \left(0.075 \times 10^{-3} \text{ m}\right)^2}{\left(5.6 \times 10^{-8} \Omega \cdot \text{m}\right) \left[1 + 4.5 \times 10^{-3} \left(\text{C}^{-1}\right)\right] \left(1320 \text{ °C} - 20.0 \text{ °C}\right)} = 3.7 \text{ m} $$

The resistivity at 20.0 °C, $\rho_0 = 5.6 \times 10^{-8} \Omega \cdot \text{m}$, was obtained from Table 20.1.
19. **SSM REASONING** We will ignore any changes in length due to thermal expansion. Although the resistance of each section changes with temperature, the total resistance of the composite does not change with temperature. Therefore,

\[
\left( R_{\text{tungsten}} \right)_0 + \left( R_{\text{carbon}} \right)_0 = \left( R_{\text{tungsten}} \right)_0 + R_{\text{carbon}} \\
\text{At room temperature} \quad \text{At temperature } T
\]

From Equation 20.5, we know that the temperature dependence of the resistance for a wire of resistance \( R_0 \) at temperature \( T_0 \) is given by \( R = R_0[1 + \alpha(T - T_0)] \), where \( \alpha \) is the temperature coefficient of resistivity. Thus,

\[
\left( R_{\text{tungsten}} \right)_0 + \left( R_{\text{carbon}} \right)_0 = \left( R_{\text{tungsten}} \right)_0 (1 + \alpha_{\text{tungsten}} \Delta T) + \left( R_{\text{carbon}} \right)_0 (1 + \alpha_{\text{carbon}} \Delta T)
\]

Since \( \Delta T \) is the same for each wire, this simplifies to

\[
\left( R_{\text{tungsten}} \right)_0 \alpha_{\text{tungsten}} = - \left( R_{\text{carbon}} \right)_0 \alpha_{\text{carbon}} \tag{1}
\]

This expression can be used to find the ratio of the resistances. Once this ratio is known, we can find the ratio of the lengths of the sections with the aid of Equation 20.3 \((L = RA/\rho)\).

**SOLUTION** From Equation (1), the ratio of the resistances of the two sections of the wire is

\[
\frac{\left( R_{\text{tungsten}} \right)_0}{\left( R_{\text{carbon}} \right)_0} = \frac{-\alpha_{\text{carbon}}}{\alpha_{\text{tungsten}}} = \frac{-0.0005 [(C^0)^{-1}]}{0.0045 [(C^0)^{-1}]} = \frac{1}{9}
\]

Thus, using Equation 20.3, we find the ratio of the tungsten and carbon lengths to be

\[
\frac{L_{\text{tungsten}}}{L_{\text{carbon}}} = \frac{\left( R_0 A/\rho \right)_{\text{tungsten}}}{\left( R_0 A/\rho \right)_{\text{carbon}}} = \frac{\left( R_{\text{tungsten}} \right)_0 \left( \rho_{\text{carbon}} \right)}{\left( R_{\text{carbon}} \right)_0 \left( \rho_{\text{tungsten}} \right)} = \frac{1}{9} \left( \frac{3.5 \times 10^{-5} \Omega \cdot \text{m}}{5.6 \times 10^{-8} \Omega \cdot \text{m}} \right) = 70
\]

where we have used resistivity values from Table 20.1 and the fact that the two sections have the same cross-sectional areas.

20. **REASONING AND SOLUTION** The voltage \( V_{\text{Cu}} \) between the ends of the copper rod is given by Ohm’s law as \( V_{\text{Cu}} = IR_{\text{Cu}} \), where \( R_{\text{Cu}} \) is the resistance of the copper rod. The current \( I \) in the circuit is equal to the voltage \( V \) of the battery that is connected across the free ends of the copper-iron rod divided by the equivalent resistance of the rod. The copper and iron rods are joined end-to-end, so the same current passes through each. Thus, they are connected in series, so the equivalent resistance \( R_S \) is \( R_S = R_{\text{Cu}} + R_{\text{Fe}} \). Thus, the current is
\[ I = \frac{V}{R_s} = \frac{V}{R_{Cu} + R_{Fe}} \]

The voltage across the copper rod is

\[ V_{Cu} = IR_{Cu} = \frac{V}{R_{Cu} + R_{Fe}} R_{Cu} \]

The resistance of the copper and iron rods is given by \( R_{Cu} = \rho_{Cu} \frac{L}{A} \) and \( R_{Fe} = \rho_{Fe} \frac{L}{A} \), where the length \( L \) and cross-sectional area \( A \) are the same for both rods and \( \rho_{Cu} \) and \( \rho_{Fe} \) denote the resistivities. Substituting these expressions for the resistances into the equation above and using resistivities from Table 20.1 yield

\[
V_{Cu} = \left( \frac{V}{\rho_{Cu} + \rho_{Fe}} \right) \rho_{Cu} = \frac{12 \text{ V}}{1.72 \times 10^{-8} \Omega \cdot \text{m} + 9.7 \times 10^{-8} \Omega \cdot \text{m}} \left( 1.72 \times 10^{-8} \Omega \cdot \text{m} \right) = 1.8 \text{ V}
\]

21. **REASONING AND SOLUTION**  The resistance of the thermistor decreases by 15% relative to its normal value of 37.0 °C. That is,

\[
\frac{\Delta R}{R_0} = \frac{R - R_0}{R_0} = -0.15
\]

According to Equation 20.5, we have

\[
R = R_0 [1 + \alpha (T - T_0)] \quad \text{or} \quad (R - R_0) = \alpha R_0 (T - T_0) \quad \text{or} \quad \frac{R - R_0}{R_0} = \alpha (T - T_0) = -0.15
\]

Rearranging this result gives

\[
T = T_0 + \frac{-0.15}{\alpha} = 37.0 \text{ °C} + \frac{-0.15}{-0.060 \text{ (°C)}^{-1}} = 39.5 \text{ °C}
\]

22. **REASONING**  Knowing the power \( P \) (140 W) consumed by the blanket and the voltage \( V \) (120 V) being applied to it, we can use \( P = \frac{V^2}{R} \) (Equation 20.6c) to determine the resistance \( R \).

**SOLUTION**  According to Equation 20.6c, we have

\[
P = \frac{V^2}{R} \quad \text{or} \quad R = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{140 \text{ W}} = 1.0 \times 10^2 \Omega
\]
23. **REASONING AND SOLUTION** According to Equation 20.6c, the power delivered to the iron is

\[ P = \frac{V^2}{R} = \frac{(120 \, \text{V})^2}{24 \, \Omega} = 6.0 \times 10^2 \, \text{W} \]

24. **REASONING AND SOLUTION** The power delivered is \( P = VI \), so that we have

a. \[ P_{bd} = VI_{bd} = (120 \, \text{V})(11 \, \text{A}) = 1300 \, \text{W} \]

b. \[ P_{vc} = VI_{vc} = (120 \, \text{V})(4.0 \, \text{A}) = 480 \, \text{W} \]

c. The energy is \( E = Pt \), so that we have

\[ \frac{E_{bd}}{E_{vc}} = \frac{P_{bd}t_{bd}}{P_{vc}t_{vc}} = \frac{(1300 \, \text{W})(15 \, \text{min})}{(480 \, \text{W})(30.0 \, \text{min})} = 1.4 \]

25. **REASONING** The total cost of keeping all the TVs turned on is equal to the number of TVs times the cost to keep each one on. The cost for one TV is equal to the energy it consumes times the cost per unit of energy ($0.12 per kW·h). The energy that a single set uses is, according to Equation 6.10b, the power it consumes times the time of use.

**SOLUTION** The total cost is

\[ \text{Total cost} = (110 \, \text{million sets}) \left( \text{Cost per set} \right) \]

\[ = (110 \, \text{million sets}) \left[ \text{Energy (in kW·h) used per set} \left( \frac{$0.12}{1 \, \text{kW·h}} \right) \right] \]

The energy (in kW·h) used per set is the product of the power and the time, where the power is expressed in kilowatts and the time is in hours:

\[ \text{Energy used per set} = Pt = (75 \, \text{W}) \left( \frac{1 \, \text{kW}}{1000 \, \text{W}} \right) (6.0 \, \text{h}) \quad \text{(6.10b)} \]

The total cost of operating the TV sets is

\[ \text{Total cost} = (110 \, \text{million sets}) \left[ (75 \, \text{W}) \left( \frac{1 \, \text{kW}}{1000 \, \text{W}} \right) (6.0 \, \text{h}) \right] \left( \frac{$0.12}{1 \, \text{kW·h}} \right) = 5.9 \times 10^6 \]
26. **REASONING** To find the current, we can use the fact that the power \( P \) is the product of the current \( I \) and the voltage \( V \), since the power and voltage are known.

**SOLUTION** Solving \( P = IV \) (Equation 20.6a) for the current, we have

\[
I = \frac{P}{V} = \frac{0.095 \text{ W}}{3.7 \text{ V}} = \boxed{0.026 \text{ A}}
\]

27. **SSM REASONING** According to Equation 6.10b, the energy used is Energy = \( Pt \), where \( P \) is the power and \( t \) is the time. According to Equation 20.6a, the power is \( P = IV \), where \( I \) is the current and \( V \) is the voltage. Thus, Energy = \( IVt \), and we apply this result first to the dryer and then to the computer.

**SOLUTION** The energy used by the dryer is

\[
\text{Energy} = Pt = IVt = (16 \text{ A})(240 \text{ V})(45 \text{ min}) \left( \frac{60 \text{ s}}{1.00 \text{ min}} \right) = 1.04 \times 10^7 \text{ J}
\]

For the computer, we have

\[
\text{Energy} = 1.04 \times 10^7 \text{ J} = IVt = (2.7 \text{ A})(120 \text{ V})t
\]

Solving for \( t \) we find

\[
t = \frac{1.04 \times 10^7 \text{ J}}{(2.7 \text{ A})(120 \text{ V})} = 3.21 \times 10^4 \text{ s} = \left( 3.21 \times 10^4 \text{ s} \right) \left( \frac{1.00 \text{ h}}{3600 \text{ s}} \right) = 8.9 \text{ h}
\]

28. **REASONING** A certain amount of time \( t \) is needed for the heater to deliver the heat \( Q \) required to raise the temperature of the water, and this time depends on the power produced by the heater. The power \( P \) is the energy (heat in this case) per unit time, so the time is the heat divided by the power or \( t = Q/P \). The heat required to raise the temperature of a mass \( m \) of water by an amount \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity of water \([4186 \text{ J/(kg·C°)}\)], see Table 12.2]. The power dissipated in a resistance \( R \) is given by Equation 20.6c as \( P = \frac{V^2}{R} \), where \( V \) is the voltage across the resistor. Using these expressions for \( Q \) and \( P \) will allow us to determine the time \( t \).

**SOLUTION** Substituting Equations 12.4 and 20.6c into the expression for the time and recognizing that the normal boiling point of water is 100.0 °C, we find that

\[
t = \frac{Q}{P} = \frac{cm\Delta T}{V^2/R} = \frac{Rcm\Delta T}{V^2}
\]

\[
= (15 \Omega) \left[ 4186 \text{ J/(kg·C°)} \right] (0.50 \text{ kg})(100.0 \text{ °C} - 13 \text{ °C}) \left( \frac{120 \text{ V}}{2} \right)^2 = 190 \text{ s}
\]
29. **REASONING** The total volume of ice melted is the product of the thickness $h$ and the area $A$ of the layer that melts, which permits us to determine the thickness in terms of the volume and the area:

$$hA = \text{Volume} \quad \text{or} \quad h = \frac{\text{Volume}}{A} \quad (1)$$

Given the density $\rho = 917 \text{ kg/m}^3$, we can find the volume of the ice from $\rho = \frac{m}{\text{Volume}}$ (Equation 11.1), where $m$ is the mass of the ice.

$$\text{Volume} = \frac{m}{\rho} \quad (2)$$

The mass $m$ of the ice determines the heat $Q$ required to melt it, according to Equation 12.5

$$Q = mL \quad (12.5)$$

In Equation 12.5, $L = 33.5 \times 10^4 \text{ J/kg}$ is the latent heat of fusion for water (see Table 12.3). In order to find the maximum thickness of ice the defroster can melt, we will assume that all the heat generated by the defroster goes into melting the ice.

The power output $P$ of the defroster is the rate at which it converts electrical energy to heat, so we have that

$$P = \frac{Q}{t} \quad \text{or} \quad Q = Pt \quad (6.10b)$$

where $t$ is the elapsed time (3.0 minutes). The power output $P$ is, in turn, found from the operating voltage $V$ and current $I$:

$$P = IV \quad (20.6a)$$

**SOLUTION** Substituting Equation (2) into Equation (1) yields

$$h = \frac{\text{Volume}}{A} = \left(\frac{m}{\rho}\right) = \frac{m}{\rho A} \quad (3)$$

Solving $Q = mL$ (Equation 12.5) for $m$, we find that $m = \frac{Q}{L}$. Substituting this result into Equation (3), we obtain

$$h = \frac{m}{\rho A} = \frac{(Q/L)}{\rho A} = \frac{Q}{L\rho A} \quad (4)$$

Substituting Equation (6.10b) into Equation (4) yields

$$h = \frac{Q}{L\rho A} = \frac{Pt}{L\rho A} \quad (5)$$
We note that the elapsed time \( t \) is given in minutes, which must be converted to seconds. Substituting Equation 20.6a into Equation (5), we obtain the maximum thickness of the ice that the defroster can melt:

\[
h = \frac{Pt}{L \rho A} = \frac{IVt}{L \rho A} = \frac{(23 \text{ A})(12 \text{ V})(3.0 \text{ min})}{(33.5 \times 10^4 \text{ J/kg})(917 \text{ kg/m}^3)(0.52 \text{ m}^2)} = 3.1 \times 10^{-4} \text{ m}
\]

30. **REASONING AND SOLUTION** We know that the resistance of the wire can be obtained from

\[
P = \frac{V^2}{R} \quad \text{or} \quad R = \frac{V^2}{P}
\]

We also know that \( R = \rho L/A \). Solving for the length, noting that \( A = \pi r^2 \), and using \( \rho = 100 \times 10^{-8} \Omega \cdot \text{m} \) from Table 20.1, we find

\[
L = \frac{RA}{\rho} = \frac{V^2}{\rho \left( \frac{\pi r^2}{\rho} \right)} = \frac{V^2 \pi r^2}{\rho^2 P} = \frac{(120 \text{ V})^2 \pi (6.5 \times 10^{-4} \text{ m})^2}{(100 \times 10^{-8} \Omega \cdot \text{m})(4.00 \times 10^2 \text{ W})} = 50 \text{ m}
\]

31. **SSM REASONING AND SOLUTION** As a function of temperature, the resistance of the wire is given by Equation 20.5: \( R = R_o \left[ 1 + \alpha (T - T_o) \right] \), where \( \alpha \) is the temperature coefficient of resistivity. From Equation 20.6c, we have \( P = V^2 / R \). Combining these two equations, we have

\[
P = \frac{V^2}{R_o \left[ 1 + \alpha (T - T_o) \right]} = \frac{P_0}{1 + \alpha (T - T_o)}
\]

where \( P_0 = V^2 / R_o \), since the voltage is constant. But \( P = \frac{1}{2} P_0 \), so we find

\[
\frac{P_0}{2} = \frac{P_0}{1 + \alpha (T - T_o)} \quad \text{or} \quad 2 = 1 + \alpha (T - T_o)
\]

Solving for \( T \), we find

\[
T = \frac{1}{\alpha} + T_o = \frac{1}{0.0045 \text{ (C}^\circ)^{-1}} + 28^\circ = 250 \text{ C}^\circ
\]

32. **REASONING** Substituting \( V = \frac{1}{2} V_0 \) into Equation 20.7 gives a result that can be solved directly for the desired time.

**SOLUTION** From Equation 20.7 we have
\[ V = \frac{1}{2} V_0 = V_0 \sin 2\pi ft \quad \text{or} \quad \frac{1}{2} = \sin 2\pi ft \]

Using the inverse trigonometric sine function, we find

\[ 2\pi ft = \sin^{-1}\left(\frac{1}{2}\right) = 0.524 \]

In this result, the value of 0.524 is in radians and corresponds to an angle of 30.0°. Thus we find that the smallest value of \( t \) is

\[ t = \frac{0.524}{2\pi f} = \frac{0.524}{2\pi(60.0 \text{ Hz})} = 1.39 \times 10^{-3} \text{ s} \]

33. **REASONING**

a. The average power \( \bar{P} \) delivered to the copy machine is equal to the square of the rms-current \( I_{\text{rms}} \) times the resistance \( R \), or \( \bar{P} = I_{\text{rms}}^2 R \) (Equation 20.15b). Both \( I_{\text{rms}} \) and \( R \) are known.

b. According to the discussion in Section 20.5, the peak power \( P_{\text{peak}} \) is twice the average power, or \( P_{\text{peak}} = 2\bar{P} \).

**SOLUTION**

a. The average power is

\[ \bar{P} = I_{\text{rms}}^2 R = (6.50 \text{ A})^2 (18.6 \text{ }\Omega) = [786 \text{ W}] \quad (20.15b) \]

b. The peak power is twice the average power, so

\[ P_{\text{peak}} = 2\bar{P} = 2(786 \text{ W}) = [1572 \text{ W}] \]

34. **REASONING** The peak voltage \( V_0 \) can be obtained from the rms voltage, since the two voltages are related according to \( V_{\text{rms}} = V_0 / \sqrt{2} \) (Equation 20.13). Knowing the rms current \( I_{\text{rms}} \) (0.50 A) and the resistance \( R \) (47 Ω), we can use \( V_{\text{rms}} = I_{\text{rms}} R \) (Ohm’s law, Equation 20.14) to determine the rms voltage \( V_{\text{rms}} \).

**SOLUTION** According to Equation 20.13, we have

\[ V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \quad \text{or} \quad V_0 = \sqrt{2}V_{\text{rms}} \]

Using \( V_{\text{rms}} = I_{\text{rms}} R \) (Ohm’s law, Equation 20.14) to substitute into the expression for the peak voltage \( V_0 \), we obtain

\[ V_0 = \sqrt{2}V_{\text{rms}} = \sqrt{2} I_{\text{rms}} R = \sqrt{2}(0.50 \text{ A})(47 \text{ Ω}) = [33 \text{ V}] \]
35. **REASONING** Because we are ignoring the effects of temperature on the heater, the resistance $R$ of the heater is the same whether it is plugged into a 120-V outlet or a 230-V outlet. The average power output $\bar{P}$ of the heater is related to the rms outlet voltage $V_{\text{rms}}$ and the heater’s resistance $R$ by $\bar{P} = \frac{V_{\text{rms}}^2}{R}$ (Equation 20.15c).

**SOLUTION** When plugged into an outlet in the US that has an rms voltage $V_{\text{rms}} = 120$ V, the average power output $\bar{P}$ of the heater is, from Equation 20.15c,

$$\bar{P} = \frac{V_{\text{rms}}^2}{R}$$

(Equation 20.15c)

When operated in Germany, the average power output ($\bar{P}_0 = 550$ W) of the heater is given by $\bar{P}_0 = \frac{V_{\text{rms},0}^2}{R}$ (Equation 20.15c), where $V_{\text{rms},0} = 230$ V. Solving this relation for $R$, we obtain

$$R = \frac{V_{\text{rms},0}^2}{\bar{P}_0}$$

(Equation 1)

Substituting Equation (1) into Equation 20.15c yields

$$\bar{P} = \frac{V_{\text{rms}}^2}{R} = \frac{V_{\text{rms}}^2}{\left(\frac{V_{\text{rms},0}^2}{\bar{P}_0}\right)} = \frac{V_{\text{rms}}^2}{V_{\text{rms},0}^2} \bar{P}_0 = \frac{(120 \text{ V})^2}{(230 \text{ V})^2} (550 \text{ W}) = 150 \text{ W}$$

36. **REASONING AND SOLUTION** The power $P$ dissipated in the extension cord is $P = I^2 R$ (Equation 20.6b). The resistance $R$ is related to the length $L$ of the wire and its cross-sectional area $A$ by Equation 20.3, $R = \rho L/A$, where $\rho$ is the resistivity of copper. The cross-sectional area of the wire can be expressed as

$$A = \frac{\rho L}{R} = \frac{\rho I^2}{P/L}$$

where the ratio $P/L$ is the power per unit length of copper wire that the heater produces. The wire is cylindrical, so its cross-sectional area is $A = \pi r^2$. Thus, the smallest radius of wire that can be used is

$$r = I \sqrt{\frac{\rho}{\pi \left(\frac{P}{L}\right)}} = (18 \text{ A}) \sqrt{\frac{1.72 \times 10^{-8} \Omega \cdot \text{m}}{\pi (1.0 \text{ W/m})}} = 1.3 \times 10^{-3} \text{ m}$$
Note that we have used 1.0 W/m as the power per unit length, rather than 2.0 W/m. This is because an extension cord is composed of two copper wires. If the maximum power per unit length that the extension cord itself can produce is 2.0 W/m, then each wire can produce only a maximum of 1.0 W/m.

37. **REASONING** The average power is given by Equation 20.15c as \( \bar{P} = \frac{V_{\text{rms}}^2}{R} \). In this expression the rms voltage \( V_{\text{rms}} \) appears. However, we seek the peak voltage \( V_0 \). The relation between the two types of voltage is given by Equation 20.13 as \( V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \), so we can obtain the peak voltage by using Equation 20.13 to substitute into Equation 20.15c.

**SOLUTION** Substituting \( V_{\text{rms}} \) from Equation 20.13 into Equation 20.15c gives

\[
\bar{P} = \frac{V_{\text{rms}}^2}{R} = \left( \frac{V_0}{\sqrt{2}} \right)^2 = \frac{V_0^2}{2R}
\]

Solving for the peak voltage \( V_0 \) gives

\[ V_0 = \sqrt{2R\bar{P}} = \sqrt{2(4.0 \, \Omega)(55 \, \text{W})} = 21 \, \text{V} \]

38. **REASONING** The higher the average power output \( \bar{P} \) of the resistance heater, the faster it can supply the heat \( Q \) needed to raise the water’s temperature. This follows from \( \bar{P} = \frac{Q}{t} \) (Equation 6.10b), which implies that the recovery time is \( t = \frac{Q}{\bar{P}} \). The average power output \( \bar{P} \) of the resistance heater is given by \( \bar{P} = \frac{V_{\text{rms}}^2}{R} \) (Equation 20.15c), where \( R \) is the resistance of the heater and \( V_{\text{rms}} = 120 \, \text{V} \) is the heater’s rms operating voltage. The heater must supply an amount of heat \( Q = cm\Delta T \) (Equation 12.4) to cause an increase \( \Delta T \) in the temperature of a mass \( m \) of water, where \( c = 4186 \, \text{J/(kg} \cdot \text{C}) \) is the specific heat capacity of water (see Table 12.2). The mass \( m \) of the water can be found from the density \( \rho = 1.000 \times 10^3 \, \text{kg/m}^3 \) (see Table 11.1) and the volume \( V_{\text{water}} \) of the water held in the heater, according to \( m = \rho V_{\text{water}} \) (Equation 11.1).

**SOLUTION** Substituting \( \bar{P} = \frac{V_{\text{rms}}^2}{R} \) (Equation 20.15c) and \( Q = cm\Delta T \) (Equation 12.4) into the expression \( t = \frac{Q}{\bar{P}} \) for the recovery time, we obtain

\[
t = \frac{Q}{\bar{P}} = \left( \frac{cm\Delta T}{\frac{V_{\text{rms}}^2}{R}} \right) = \frac{Rcm\Delta T}{V_{\text{rms}}^2}
\]
Substituting \( m = \rho V_{\text{water}} \) (Equation 11.1) into Equation (1) yields

\[
t = \frac{R c m \Delta T}{V_{\text{rms}}^2} = \frac{R c \rho V_{\text{water}} \Delta T}{V_{\text{rms}}^2} \quad (2)
\]

Before using Equation (2) to calculate the recovery time, we must convert the volume \( V_{\text{water}} \) of the water in the unit from gallons to \( \text{m}^3 \), using the equivalence 1 gal = \( 3.79 \times 10^{-3} \) \( \text{m}^3 \):

\[
V_{\text{water}} = (52 \text{ gal}) \left( \frac{3.79 \times 10^{-3} \text{ m}^3}{1.00 \text{ gal}} \right) = 0.20 \text{ m}^3
\]

We note that the recovery time is to be expressed in hours, where 1.00 h = 3600 s. Therefore, from Equation (2), we find that

\[
t = \frac{R c \rho V_{\text{water}} \Delta T}{V_{\text{rms}}^2} = \frac{(3.0 \Omega) \left[ 4186 \frac{\text{J}}{(\text{kg} \cdot \text{C}^2)} \right] (1.000 \times 10^3 \text{ kg/m}^3)(0.20 \text{ m}^3)(53 \degree \text{C} - 11 \degree \text{C})}{(120 \text{ V})^2}
\]

\[
= 7300 \text{ s} = (7300 \text{ s}) \left( \frac{1.00 \text{ h}}{3600 \text{ s}} \right) = 2.0 \text{ h}
\]

---

39. **SSM REASONING**

a. We can obtain the frequency of the alternating current by comparing this specific expression for the current with the more general one in Equation 20.8.

b. The resistance of the light bulb is, according to Equation 20.14, equal to the rms-voltage divided by the rms-current. The rms-voltage is given, and we can obtain the rms-current by dividing the peak current by \( \sqrt{2} \), as expressed by Equation 20.12.

c. The average power is given by Equation 20.15a as the product of the rms-current and the rms-voltage.

**SOLUTION**

a. By comparing \( I = (0.707 \text{ A}) \sin \left[ (314 \text{ Hz}) t \right] \) with the general expression (see Equation 20.8) for the current in an ac circuit, \( I = I_0 \sin 2\pi f t \), we see that

\[
2\pi f t = (314 \text{ Hz}) t \quad \text{or} \quad f = \frac{314 \text{ Hz}}{2\pi} = 50.0 \text{ Hz}
\]

b. The resistance is equal to \( V_{\text{rms}}/I_{\text{rms}} \), where the rms-current is related to the peak current \( I_0 \) by \( I_{\text{rms}} = I_0 / \sqrt{2} \). Thus, the resistance of the light bulb is

\[
R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{V_{\text{rms}}}{\frac{I_0}{\sqrt{2}}} = \frac{\sqrt{2} (120.0 \text{ V})}{0.707 \text{ A}} = 2.40 \times 10^2 \Omega
\]

(20.14)
c. The average power is the product of the rms-current and rms-voltage:

\[
\bar{P} = I_{\text{rms}} V_{\text{rms}} = \left( \frac{I_0}{\sqrt{2}} \right) V_{\text{rms}} = \left( \frac{0.707 \text{ A}}{\sqrt{2}} \right)(120.0 \text{ V}) = 60.0 \text{ W}
\]  

(20.15a)

40. **REASONING AND SOLUTION** The energy \( Q_1 \) that is released when the water cools from an initial temperature \( T \) to a final temperature of 0.0 °C is given by Equation 12.4 as \( Q_1 = cm(T - 0.0 \text{ °C}) \). The energy \( Q_2 \) released when the water turns into ice at 0.0 °C is \( Q_2 = mL_f \) where \( L_f \) is the latent heat of fusion for water. Since power \( P \) is energy divided by time, the power produced is

\[
P = \frac{Q_1 + Q_2}{t} = \frac{cm(T - 0.0 \text{ °C}) + mL_f}{t}
\]

The power produced by an electric heater is, according to Equation 20.6a, \( P = IV \). Substituting this expression for \( P \) into the equation above and solving for the current \( I \), we get

\[
I = \frac{cm(T - 0.0 \text{ °C}) + mL_f}{tV}
\]

\[
I = \frac{(4186 \text{ J/kg} \cdot \text{°C})(660 \text{ kg})(10.0 \text{ °C}) + (660 \text{ kg})(33.5 \times 10^4 \text{ J/kg})}{(9.0 \text{ h})(3600 \text{ s/h})(240 \text{ V})} = 32 \text{ A}
\]

41. **SSM REASONING** The equivalent series resistance \( R_s \) is the sum of the resistances of the three resistors. The potential difference \( V \) can be determined from Ohm's law according to \( V = IR_s \).

**SOLUTION**

a. The equivalent resistance is

\[
R_s = 25 \Omega + 45 \Omega + 75 \Omega = 145 \Omega
\]

b. The potential difference across the three resistors is

\[
V = IR_s = (0.51 \text{ A})(145\Omega) = 74 \text{ V}
\]

42. **REASONING** According to Equation 20.2, the resistance \( R \) of the resistor is equal to the voltage \( V_R \) across it divided by the current \( I \), or \( R = \frac{V_R}{I} \). Since the resistor, the lamp, and the voltage source are in series, the voltage across the resistor is \( V_R = 120.0 \text{ V} - V_L \), where \( V_L \) is the voltage across the lamp. Thus, the resistance is
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1023

L

120.0 V

R

I

Since $V_L$ is known, we need only determine the current in the circuit. Since we know the voltage $V_L$ across the lamp and the power $P$ dissipated by it, we can use Equation 20.6a to find the current: $I = P/V_L$. The resistance can be written as

\[ R = \frac{120.0 \text{ V} - V_L}{P} \]

**SOLUTION** Substituting the known values for $V_L$ and $P$ into the equation above, the resistance is

\[ R = \frac{120.0 \text{ V} - 25 \text{ V}}{60.0 \text{ W}} = \frac{4.0 \times 10^3 \Omega}{25 \text{ V}} \]

43. **SSM REASONING** Using Ohm's law (Equation 20.2) we can write an expression for the voltage across the original circuit as $V = I_0R_0$. When the additional resistor $R$ is inserted in series, assuming that the battery remains the same, the voltage across the new combination is given by $V = I(R + R_0)$. Since $V$ is the same in both cases, we can write $I_0R_0 = I(R + R_0)$. This expression can be solved for $R_0$.

**SOLUTION** Solving for $R_0$, we have

\[ I_0R_0 - IR_0 = IR \quad \text{or} \quad R_0(I_0 - I) = IR \]

Therefore, we find that

\[ R_0 = \frac{IR}{I_0 - I} = \frac{(12.0 \text{ A})(8.00 \Omega)}{15.0 \text{ A} - 12.0 \text{ A}} = 32 \Omega \]

44. **REASONING** The power $P_n$ delivered to any one of the three resistors is equal to the product of the current squared ($I^2$) and the resistance $R_n$, or $P_n = I^2 R_n$, where $n = 1, 2, \text{ or } 3$. In each case, the resistance is known, and Ohm’s law can be used to find the current. Ohm’s law states that the current in the circuit (which is also the current through each of the resistors) equals the voltage $V$ of the battery divided by the equivalent resistance $R_S$ of the three resistors: $I = V/R_S$. Since the resistors are connected in series, we can obtain the equivalent resistance by adding the three resistances.
**SOLUTION** The power $P_n$ supplied to any one of the three resistors is

$$P_n = I^2 R_n \quad (n = 1, \ 2, \text{ or } 3)$$  \hspace{1cm} (20.6b)

The current $I$ depends on the voltage $V$ of the battery and the equivalent resistance $R_S$ of the three resistors through Ohm’s law:

$$I = \frac{V}{R_S}$$ \hspace{1cm} (20.2)

Substituting Equation 20.2 into Equation 20.6b gives

$$P_n = I^2 R_n = \left(\frac{V}{R_S}\right)^2 R_n \quad (n = 1, \ 2, \text{ or } 3)$$ \hspace{1cm} (1)

Since the three resistors are wired in series, the equivalent resistance $R_S$ is the sum of the resistances: $R_S = R_1 + R_2 + R_3$ (Equation 20.16). Substituting this relation into Equation (1) yields

$$P_n = \left(\frac{V}{R_S}\right)^2 R_n = \left(\frac{V}{R_1 + R_2 + R_3}\right)^2 R_n \quad (n = 1, \ 2, \text{ or } 3)$$

The power delivered to each resistor is:

$$P_1 = \left(\frac{V}{R_1 + R_2 + R_3}\right)^2 R_1 = \left(\frac{24 \ \text{V}}{2.0 \ \Omega + 4.0 \ \Omega + 6.0 \ \Omega}\right)^2 (2.0 \ \Omega) = 8.0 \ \text{W}$$

$$P_2 = \left(\frac{V}{R_1 + R_2 + R_3}\right)^2 R_2 = \left(\frac{24 \ \text{V}}{2.0 \ \Omega + 4.0 \ \Omega + 6.0 \ \Omega}\right)^2 (4.0 \ \Omega) = 16 \ \text{W}$$

$$P_3 = \left(\frac{V}{R_1 + R_2 + R_3}\right)^2 R_3 = \left(\frac{24 \ \text{V}}{2.0 \ \Omega + 4.0 \ \Omega + 6.0 \ \Omega}\right)^2 (6.0 \ \Omega) = 24 \ \text{W}$$

---

45. **REASONING** Since the two resistors are connected in series, they are equivalent to a single equivalent resistance that is the sum of the two resistances, according to Equation 20.16. Ohm’s law (Equation 20.2) can be applied with this equivalent resistance to give the battery voltage.

**SOLUTION** According to Ohm’s law, we find

$$V = IR_S = I \left( R_1 + R_2 \right) = (0.12 \ \text{A})(47 \ \Omega + 28 \ \Omega) = 9.0 \ \text{V}$$
46. **REASONING** The circuit containing the light bulb and resistor is shown in the drawing. The resistance $R_1$ of the light bulb is related to the power delivered to it by $R_1 = P_1 / I^2$ (Equation 20.6b), where $I$ is the current in the circuit. The power is known, and the current can be obtained from Ohm’s law as the voltage $V$ of the source divided by the equivalent resistance $R_S$ of the series circuit: $I = V / R_S$. Since the two resistors are wired in series, the equivalent resistance is the sum of the resistances, or $R_S = R_1 + R_2$.

![Circuit Diagram](image)

**SOLUTION** The resistance of the light bulb is

$$R_1 = \frac{P_1}{I^2} \quad (20.6b)$$

Substituting $I = V / R_S$ (Equation 20.2) into Equation 20.6b gives

$$R_1 = \frac{P_1}{I^2} = \frac{P_1}{(V / R_S)^2} = \frac{P_1 R_S^2}{V^2} \quad (1)$$

The equivalent resistance of the two resistors wired in series is $R_S = R_1 + R_2$ (Equation 20.16). Substituting this expression for $R_S$ into Equation (1) yields

$$R_1 = \frac{P_1 R_S^2}{V^2} = \frac{P_1 (R_1 + R_2)^2}{V^2}$$

Algebraically rearranging this equation, we find that

$$R_1^2 + \left( 2R_2 - \frac{V^2}{P_1} \right) R_1 + R_2^2 = 0$$

This is a quadratic equation in the variable $R_1$. The solution can be found by using the quadratic formula (see Appendix C.4):
\[
R_1 = \frac{-\left(2R_2 - \frac{V^2}{P_1}\right) \pm \sqrt{\left(2R_2 - \frac{V^2}{P_1}\right)^2 - 4R_2^2}}{2}
\]

\[
= -\left[2(144 \, \Omega) - \frac{(120.0 \, \text{V})^2}{23.4 \, \text{W}}\right] \pm \sqrt{\left(2(144 \, \Omega) - \frac{(120.0 \, \text{V})^2}{23.4 \, \text{W}}\right)^2 - 4(144 \, \Omega)^2}
\]

\[
= 85.9 \, \Omega \quad \text{and} \quad 242 \, \Omega
\]

47. **REASONING**

a. The greatest voltage for the battery is the voltage that generates the maximum current \( I \) that the circuit can tolerate. Once this maximum current is known, the voltage can be calculated according to Ohm’s law, as the current times the equivalent circuit resistance for the three resistors in series. To determine the maximum current we note that the power \( P \) dissipated in each resistance \( R \) is \( P = I^2 R \) according to Equation 20.6b. Since the power rating and resistance are known for each resistor, the maximum current that can be tolerated by a resistor is \( I = \sqrt{\frac{P}{R}} \). By examining this maximum current for each resistor, we will be able to identify the maximum current that the circuit can tolerate.

b. The battery delivers power to the circuit that is given by the battery voltage times the current, according to Equation 20.6a.

**SOLUTION**

a. Solving Equation 20.6b for the current, we find that the maximum current for each resistor is as follows:

\[
I = \frac{P}{R} = \frac{4.0 \, \text{W}}{2.0 \, \Omega} = 2.0 \, \Omega \quad I = \frac{10.0 \, \text{W}}{12.0 \, \Omega} = 0.833 \, \Omega \quad I = \frac{5.0 \, \text{W}}{3.0 \, \Omega} = 1.67 \, \Omega
\]

The smallest of these three values is 0.913 A and is the maximum current that the circuit can tolerate. Since the resistors are connected in series, the equivalent resistance of the circuit is

\[
R_S = 2.0 \, \Omega + 12.0 \, \Omega + 3.0 \, \Omega = 17.0 \, \Omega
\]

Using Ohm’s law with this equivalent resistance and the maximum current of 0.913 A reveals that the maximum battery voltage is

\[
V = IR_S = (0.913 \, \text{A})(17.0 \, \Omega) = 15.5 \, \text{V}
\]

b. The power delivered by the battery in part (a) is given by Equation 20.6a as

\[
P = IV = (0.913 \, \text{A})(15.5 \, \text{V}) = 14.2 \, \text{W}
\]
48. **REASONING** The answer is not 340 W + 240 W = 580 W. The reason is that each heater contributes resistance to the circuit when they are connected in series across the battery. For a series connection, the resistances add together to give the equivalent total resistance, according to Equation 20.16. Thus, the total resistance is greater than the resistance of either heater. The greater resistance means that the current from the battery is less than when either heater is present by itself. Since the power for each heater is \( P = I^2R \), according to Equation 20.6b, the smaller current means that the power delivered to an individual heater is less when both are connected than when that heater is connected alone. We approach this problem by remembering that the total power delivered to the series combination of the heaters is the power delivered to the equivalent series resistance.

**SOLUTION** Let the resistances of the two heaters be \( R_1 \) and \( R_2 \). Correspondingly, the powers delivered to the heaters when each is connected alone to the battery are \( P_1 \) and \( P_2 \). For the series connection, the equivalent total resistance is \( R_1 + R_2 \), according to Equation 20.16. Using Equation 20.6c, we can write the total power delivered to this equivalent resistance as

\[
P = \frac{V^2}{R_1 + R_2}
\]

But according to Equation 20.6c, as applied to the situations when each heater is connected by itself to the battery, we have

\[
P_1 = \frac{V^2}{R_1} \quad \text{or} \quad R_1 = \frac{V^2}{P_1}
\]

\[
P_2 = \frac{V^2}{R_2} \quad \text{or} \quad R_2 = \frac{V^2}{P_2}
\]

Substituting Equations (2) and (3) into Equation (1) gives

\[
P = \frac{V^2}{P_1 + \frac{V^2}{P_2}} = \frac{1}{\frac{1}{P_1} + \frac{1}{P_2}} = \frac{P_1P_2}{P_1 + P_2} = \frac{(340 \text{ W})(240 \text{ W})}{340 \text{ W} + 240 \text{ W}} = 140 \text{ W}
\]

49. **REASONING** Ohm’s law provides the basis for our solution. We will use it to express the current from the battery when both resistors are connected and when only one resistor at a time is connected. When both resistors are connected, we will use Ohm’s law with the series equivalent resistance, which is \( R_1 + R_2 \), according to Equation 20.16. The problem statement gives values for amounts by which the current increases when one or the other resistor is removed. Thus, we will focus attention on the difference between the currents given by Ohm’s law.

**SOLUTION** When \( R_2 \) is removed, leaving only \( R_1 \) connected, the current increases by 0.20 A. In this case, using Ohm’s law to express the currents, we have
\[ \frac{V}{R_1} - \frac{V}{R_1 + R_2} = \frac{VR_2}{R_1(R_1 + R_2)} = 0.20 \text{ A} \]  

\[ \frac{V}{R_2} - \frac{V}{R_1 + R_2} = \frac{VR_1}{R_2(R_1 + R_2)} = 0.10 \text{ A} \]

When \( R_1 \) is removed, leaving only \( R_2 \) connected, the current increases by 0.10 A. In this case, using Ohm's law to express the currents, we have

\[ \frac{V}{R_2} = \frac{VR_1}{R_2(R_1 + R_2)} = 0.10 \text{ A} \]

Multiplying Equation (1) and Equation (2), we obtain

\[ \left[ \frac{VR_2}{R_1(R_1 + R_2)} \right] \left[ \frac{VR_1}{R_2(R_1 + R_2)} \right] = (0.20 \text{ A})(0.10 \text{ A}) \]

Simplifying this result algebraically shows that

\[ \frac{V^2}{(R_1 + R_2)^2} = (0.20 \text{ A})(0.10 \text{ A}) \quad \text{or} \quad \frac{V}{R_1 + R_2} = \sqrt{(0.20 \text{ A})(0.10 \text{ A})} = 0.14 \text{ A} \] (3)

\[ \frac{V}{R_1} = 0.14 \text{ A} = 0.20 \text{ A} \quad \text{or} \quad R_1 = \frac{V}{0.20 \text{ A} + 0.14 \text{ A}} = \frac{12 \text{ V}}{0.20 \text{ A} + 0.14 \text{ A}} = 35 \Omega \]

\[ \frac{V}{R_2} = 0.14 \text{ A} = 0.10 \text{ A} \quad \text{or} \quad R_2 = \frac{V}{0.10 \text{ A} + 0.14 \text{ A}} = \frac{12 \text{ V}}{0.10 \text{ A} + 0.14 \text{ A}} = 5.0 \times 10^1 \Omega \]

### 50. REASONING
The total power \( P \) is given by \( P = \frac{V^2}{R_p} \) (Equation 20.6c), where \( V \) is the outlet voltage (120 V) and \( R_p \) is the equivalent parallel resistance of the coffee-maker and the toaster. The equivalent parallel resistance can be determined with the aid of the following expression:

\[ \frac{1}{R_p} = \frac{1}{R_{\text{coffee-maker}}} + \frac{1}{R_{\text{toaster}}} \] (Equation 20.17).

### SOLUTION
Using Equation 20.6c for the total power and Equation 20.17 to deal with the equivalent parallel resistance of the two appliances, we have
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51. **REASONING AND SOLUTION** The power $P$ dissipated in a resistance $R$ is given by Equation 20.6c as $P = V^2 / R$. The resistance $R_{50}$ of the 50.0-W filament is

\[
R_{50} = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{50.0 \text{ W}} = 288 \Omega
\]

The resistance $R_{100}$ of the 100.0-W filament is

\[
R_{100} = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{100.0 \text{ W}} = 144 \Omega
\]

52. **REASONING**

a. The three resistors are in series, so the same current goes through each resistor: $I_1 = I_2 = I_3$. The voltage across each resistor is given by Equation 20.2 as $V = IR$. Because the current through each resistor is the same, the voltage across each is proportional to the resistance. Since $R_1 > R_2 > R_3$, we expect the ranking of the voltages to be $V_1 > V_2 > V_3$.

b. The three resistors are in parallel, so the same voltage exists across each: $V_1 = V_2 = V_3$. The current through each resistor is given by Equation 20.2 as $I = V/R$. Because the voltage across each resistor is the same, the current through each is inversely proportional to the resistance. Since $R_1 > R_2 > R_3$, we expect the ranking of the currents to be $I_3 > I_2 > I_1$.

**SOLUTION**

a. The current through the three resistors is given by $I = V/R_s$, where $R_s$ is the equivalent resistance of the series circuit. From Equation 20.16, the equivalent resistance is $R_s = 50.0 \Omega + 25.0 \Omega + 10.0 \Omega = 85.0 \Omega$. The current through each resistor is

\[
I_1 = I_2 = I_3 = \frac{V}{R_s} = \frac{24.0 \text{ V}}{85.0 \Omega} = 0.282 \text{ A}
\]

The voltage across each resistor is

\[
V_1 = IR_1 = (0.282 \text{ A})(50.0 \Omega) = 14.1 \text{ V}
\]

\[
V_2 = IR_2 = (0.282 \text{ A})(25.0 \Omega) = 7.05 \text{ V}
\]

\[
V_3 = IR_3 = (0.282 \text{ A})(10.0 \Omega) = 2.82 \text{ V}
\]
b. The resistors are in parallel, so the voltage across each is the same as the voltage of the battery:

\[ V_1 = V_2 = V_3 = 24.0 \text{ V} \]

The current through each resistor is equal to the voltage across each divided by the resistance:

\[ I_1 = \frac{V}{R_1} = \frac{24.0 \text{ V}}{50.0 \Omega} = 0.480 \text{ A} \]

\[ I_2 = \frac{V}{R_2} = \frac{24.0 \text{ V}}{25.0 \Omega} = 0.960 \text{ A} \]

\[ I_3 = \frac{V}{R_3} = \frac{24.0 \text{ V}}{10.0 \Omega} = 2.40 \text{ A} \]

53. **SSM REASONING** When the switch is open, no current goes to the resistor \( R_2 \). Current exists only in \( R_1 \), so it is the equivalent resistance. When the switch is closed, current is sent to both resistors. Since they are wired in parallel, we can use Equation 20.17 to find the equivalent resistance. Whether the switch is open or closed, the power \( P \) delivered to the circuit can be found from the relation \( P = \frac{V^2}{R} \) (Equation 20.6c), where \( V \) is the battery voltage and \( R \) is the equivalent resistance.

**SOLUTION**

a. When the switch is open, there is current only in resistor \( R_1 \). Thus, the equivalent resistance is \( R_1 = 65.0 \Omega \).

b. When the switch is closed, there is current in both resistors and, furthermore, they are wired in parallel. The equivalent resistance is

\[
\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{65.0 \Omega} + \frac{1}{96.0 \Omega} \quad \text{or} \quad R_p = \frac{38.8 \Omega}{\text{(20.17)}}
\]

c. When the switch is open, the power delivered to the circuit by the battery is given by \( P = \frac{V^2}{R_1} \), since the only resistance in the circuit is \( R_1 \). Thus, the power is

\[
P = \frac{V^2}{R_1} = \frac{(9.00 \text{ V})^2}{65.0 \Omega} = 1.25 \text{ W} \quad \text{(20.6)}
\]

d. When the switch is closed, the power delivered to the circuit is \( P = \frac{V^2}{R_p} \), where \( R_p \) is the equivalent resistance of the two resistors wired in parallel:

\[
P = \frac{V^2}{R_p} = \frac{(9.00 \text{ V})^2}{38.8 \Omega} = 2.09 \text{ W} \quad \text{(20.6)}
\]
54. **REASONING** The equivalent parallel resistance $R_p$ can be determined with the aid of the following expression:

$$\frac{1}{R_p} = \frac{1}{R_{16 \ \Omega \ \text{speaker}}} + \frac{1}{R_{8 \ \Omega \ \text{speaker}}} + \frac{1}{R_{4 \ \Omega \ \text{speaker}}} \quad (\text{Equation 20.17}).$$

**SOLUTION** Using Equation 20.17, we find that

$$\frac{1}{R_p} = \frac{1}{16 \ \Omega} + \frac{1}{8 \ \Omega} + \frac{1}{4 \ \Omega} = 0.437 \ \Omega^{-1}$$

$$R_p = \frac{1}{0.437 \ \Omega^{-1}} = 2.3 \ \Omega$$

Note that we have carried an extra significant figure in determining $1/R_p$ and rounded off to the correct number of significant figures in determining $R_p$.

55. **SSM** **REASONING** Since the resistors are connected in parallel, the voltage across each one is the same and can be calculated from Ohm's Law (Equation 20.2: $V = IR$). Once the voltage across each resistor is known, Ohm's law can again be used to find the current in the second resistor. The total power consumed by the parallel combination can be found calculating the power consumed by each resistor from Equation 20.6b: $P = I^2 R$. Then, the total power consumed is the sum of the power consumed by each resistor.

**SOLUTION** Using data for the second resistor, the voltage across the resistors is equal to

$$V = IR = (3.00 \ A)(64.0 \ \Omega) = 192 \ V$$

a. The current through the 42.0-$\Omega$ resistor is

$$I = \frac{V}{R} = \frac{192 \ V}{42.0 \ \Omega} = 4.57 \ A$$

b. The power consumed by the 42.0-$\Omega$ resistor is

$$P = I^2 R = (4.57 \ A)^2 (420 \ \Omega) = 877 \ W$$

while the power consumed by the 64.0-$\Omega$ resistor is

$$P = I^2 R = (3.00 \ A)^2 (64.0 \ \Omega) = 576 \ W$$

Therefore the total power consumed by the two resistors is $877 \ W + 576 \ W = 1450 \ W$.

56. **REASONING**

a. The two identical resistors, each with a resistance $R$, are connected in parallel across the battery, so the potential difference across each resistor is $V$, the potential difference provided by the battery. The power $P$ supplied to each resistor, then, is $P = \frac{V^2}{R}$ (Equation 20.6c). The total power supplied by the battery is twice this amount.
b. According to Equation 20.6c, the resistor which is heated until its resistance is $2R$ consumes only half as much power as it did initially. Because the resistance of the other resistor does not change, it consumes the same power as before. The battery supplies less power, therefore, than it did initially. We will use Equation 20.6c to determine the final power supplied.

**SOLUTION**

a. Since the power supplied to each resistor is $P = \frac{V^2}{R}$ (Equation 20.6c), the total power is twice as large: $P_{\text{tot}} = 2P = \frac{2V^2}{R}$. Solving for $R$, we obtain

$$R = \frac{2V^2}{P_{\text{tot}}} = \frac{2(25 \text{ V})^2}{9.6 \text{ W}} = 130 \Omega$$

b. The resistances of the resistors are now $R_1 = R$ and $R_2 = 2R$. From Equation 20.6c, the total power output $P_{\text{tot}} = P_1 + P_2$ of the resistors is

$$P = P_1 + P_2 = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \frac{V^2}{R} + \frac{V^2}{2R} = \frac{3V^2}{2(130 \Omega)} = 7.2 \text{ W}$$

57. **REASONING** The total power is given by Equation 20.15c as $\bar{P} = \frac{V_{\text{rms}}^2}{R_p}$, where $R_p$ is the equivalent parallel resistance of the heater and the lamp. Since the total power and the rms voltage are known, we can use this expression to obtain the equivalent parallel resistance. This equivalent resistance is related to the individual resistances of the heater and the lamp via Equation 20.17, which is $R_p^{-1} = R_{\text{heater}}^{-1} + R_{\text{lamp}}^{-1}$. Since $R_{\text{heater}}$ is given, $R_{\text{lamp}}$ can be found once $R_p$ is known.

**SOLUTION** According to Equation 20.15c, the equivalent parallel resistance is

$$R_p = \frac{V_{\text{rms}}^2}{\bar{P}}$$

Using this result in Equation 20.17 gives

$$\frac{1}{R_p} = \frac{V_{\text{rms}}^2}{\bar{P}} = \frac{1}{R_{\text{heater}}} + \frac{1}{R_{\text{lamp}}}$$

Rearranging this expression shows that

$$\frac{1}{R_{\text{lamp}}} = \frac{\bar{P}}{V_{\text{rms}}^2} - \frac{1}{R_{\text{heater}}} = \frac{111 \text{ W}}{(120 \text{ V})^2} - \frac{1}{4.0 \times 10^2 \Omega} = 5.2 \times 10^{-3} \Omega^{-1}$$

Therefore,

$$R_{\text{lamp}} = \frac{1}{5.2 \times 10^{-3} \Omega^{-1}} = 190 \Omega$$
58. **REASONING** The series combination has an equivalent resistance of \( R_S = R_1 + R_2 \), as given by Equation 20.16. The parallel combination has an equivalent resistance that can be determined from \( R_p^{-1} = R_1^{-1} + R_2^{-1} \), according to Equation 20.17. In each case the equivalent resistance can be used in Ohm’s law with the given voltage and current. Thus, we can obtain two equations that each contain the unknown resistances. These equations will be solved simultaneously to obtain \( R_1 \) and \( R_2 \).

**SOLUTION** For the series case, Ohm’s law is \( V = I_S (R_1 + R_2) \). Solving for the sum of the resistances, we have

\[
R_1 + R_2 = \frac{V}{I_S} = \frac{12.0 \text{ V}}{2.00 \text{ A}} = 6.00 \Omega
\]  

(1)

For the parallel case, Ohm’s law is \( V = I_P R_p \), where \( R_p^{-1} = R_1^{-1} + R_2^{-1} \). Thus, we have

\[
\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{I_P}{V} = \frac{9.00 \text{ A}}{12.0 \text{ V}} = 0.750 \Omega^{-1}
\]  

(2)

Solving Equation (1) for \( R_2 \) and substituting the result into Equation (2) gives

\[
\frac{1}{R_1} + \frac{1}{6.00 - R_1} = 0.750 \quad \text{or} \quad R_1^2 - 6.00R_1 + 8.00 = 0
\]

In this result we have suppressed the units in the interest of clarity. Solving the quadratic equation (see Appendix C.4 for the quadratic formula) gives

\[
R_1 = \frac{-(-6.00) \pm \sqrt{(-6.00)^2 - 4(1.00)(8.00)}}{2(1.00)} = \frac{6.00 \pm \sqrt{36.0 - 32.0}}{2.00} = 4.00 \text{ or } 2.00
\]

Substituting these values for \( R_1 \) into Equation (1) reveals that

\[
R_2 = 6.00 - R_1 = 2.00 \text{ or } 4.00
\]

Thus, the values for the two resistances are \( 2.00 \Omega \text{ and } 4.00 \Omega \).

59. **REASONING AND SOLUTION** The aluminum and copper portions may be viewed as being connected in parallel since the same voltage appears across them. Using \( a \) and \( b \) to denote the inner and outer radii, respectively, and using Equation 20.3 to express the resistance for each portion, we find for the equivalent resistance that
We have taken resistivity values for copper and aluminum from Table 20.1:

\[
\begin{array}{rll}
\frac{1}{R_p} &= \frac{1}{R_{Al}} + \frac{1}{R_{Cu}} = \frac{A_{Cu}}{\rho_{Cu}L} + \frac{A_{Al}}{\rho_{Al}L} = \frac{\pi a^2}{\rho_{Cu}L} + \frac{\pi (b^2-a^2)}{\rho_{Al}L} \\
&= \frac{\pi \left(2.00 \times 10^{-3} \text{ m}\right)^2}{\left(1.72 \times 10^{-8} \text{ } \Omega \cdot \text{m}\right)(1.50 \text{ m})} + \frac{\pi \left[\left(3.00 \times 10^{-3} \text{ m}\right)^2 - \left(2.00 \times 10^{-3} \text{ m}\right)^2\right]}{\left(2.82 \times 10^{-8} \text{ } \Omega \cdot \text{m}\right)(1.50 \text{ m})} = 0.00116 \text{ } \Omega
\end{array}
\]

60. **REASONING** The total power \( P \) delivered by the battery is related to the equivalent resistance \( R_{eq} \) connected between the battery terminals and to the battery voltage \( V \) according to Equation 20.6c: \( P = \frac{V^2}{R_{eq}} \).

When two resistors are connected in series, the equivalent resistance \( R_S \) of the combination is greater than the resistance of either resistor alone. This can be seen directly from \( R_S = R_1 + R_2 \) (Equation 20.16).

When two resistors are connected in parallel, the equivalent resistance \( R_P \) of the combination is smaller than the resistance of either resistor alone. This can be seen directly by substituting values in \( R_P^{-1} = R_1^{-1} + R_2^{-1} \) (Equation 20.17) or by reviewing the discussion in Section 20.7 concerning the water flow analogy for electric current in a circuit.

Since the total power delivered by the battery is \( P = \frac{V^2}{R_{eq}} \) and since the power and the battery voltage are the same in both cases, it follows that the equivalent resistances are also the same. But the parallel combination has an equivalent resistance \( R_P \) that is smaller than \( R_B \), whereas the series combination has an equivalent resistance \( R_S \) that is greater than \( R_A \). This means that \( R_B \) must be greater than \( R_A \), as Diagram 1 at the right shows. If \( R_A \) were greater than \( R_B \), as in Diagram 2, the equivalent resistances \( R_S \) and \( R_P \) would not be equal.

**SOLUTION** As discussed in the **REASONING**, the equivalent resistances in circuits A and B are equal. According to Equations 20.16 and 20.17, the series and parallel equivalent resistances are
\[ R_S = R_A + R_A = 2R_A \]
\[ \frac{1}{R_p} = \frac{1}{R_B} + \frac{1}{R_B} \quad \text{or} \quad R_p = \frac{1}{2} R_B \]

Setting the equivalent resistances equal gives
\[ 2R_A = \frac{1}{2} R_B \quad \text{or} \quad \frac{R_B}{R_A} = 4 \]

As expected, \( R_B \) is greater than \( R_A \).

61. **REASONING** Since the defogger wires are connected in parallel, the total resistance of all thirteen wires can be obtained from Equation 20.17:
\[ \frac{1}{R_p} = \frac{13}{R} \quad \text{or} \quad R_p = \frac{R}{13} \]

where \( R \) is the individual resistance of one of the wires. The heat required to melt the ice is given by \( Q = mL_f \), where \( m \) is the mass of the ice and \( L_f \) is the latent heat of fusion of the ice (see Section 12.8). Therefore, using Equation 20.6c, we can see that the power or energy dissipated per unit time in the wires and used to melt the ice is
\[ P = \frac{V^2}{R_p} = \frac{mL_f}{t} \quad \text{or} \quad \frac{V^2}{R/13} = \frac{mL_f}{t} \]

According to Equation 20.3, \( R = \rho L/A \), where the length of the wire is \( L \), its cross-sectional area is \( A \) and its resistivity is \( \rho \); therefore, the last expression can be written
\[ \frac{13V^2}{R} = \frac{13V^2}{\rho L/A} = \frac{mL_f}{t} \]

This expression can be solved for the area \( A \).

**SOLUTION** Solving the above expression for \( A \), and substituting given data, and obtaining the latent heat of fusion of water from Table 12.3, we find that
\[ A = \frac{\rho L m L_f}{13 V^2 t} \]
\[ = \frac{(88.0 \times 10^{-8} \ \Omega \cdot m)(1.30 \ m)(2.10 \times 10^{-2} \ kg)(33.5 \times 10^4 \ J/kg)}{13(12.0 \ V)^2(120 \ s)} = 3.58 \times 10^{-8} \ m^2 \]
62. **REASONING** The circuit diagram is shown at the right. We can find the current in the 120.0-Ω resistor by using Ohm’s law, provided that we can obtain a value for $V_{AB}$, the voltage between points A and B in the diagram. To find $V_{AB}$, we will also apply Ohm’s law, this time by multiplying the current from the battery times $R_{AB}$, the equivalent parallel resistance between A and B. The current from the battery can be obtained by applying Ohm’s law again, now to the entire circuit and using the total equivalent resistance of the series combination of the 20.0-Ω resistor and $R_{AB}$. Once the current in the 120.0-Ω resistor is found, the power delivered to it can be obtained from Equation 20.6b, $P = I^2 R$.

**SOLUTION**
a. According to Ohm’s law, the current in the 120.0-Ω resistor is $I_{120} = V_{AB}/(120.0 \ \Omega)$. To find $V_{AB}$, we note that the equivalent parallel resistance between points A and B can be obtained from Equation 20.17 as follows:

$$\frac{1}{R_{AB}} = \frac{1}{60.0 \ \Omega} + \frac{1}{120.0 \ \Omega} \quad \text{or} \quad R_{AB} = 40.0 \ \Omega$$

This resistance of 40.0 Ω is in series with the 20.0-Ω resistance, so that according to Equation 20.16, the total equivalent resistance connected across the battery is $40.0 \ \Omega + 20.0 \ \Omega = 60.0 \ \Omega$. Applying Ohm’s law to the entire circuit, we can see that the current from the battery is

$$I = \frac{15.0 \ \text{V}}{60.0 \ \Omega} = 0.250 \ \text{A}$$

Again applying Ohm’s law, this time to the resistance $R_{AB}$, we find that

$$V_{AB} = (0.250 \ \text{A})R_{AB} = (0.250 \ \text{A})(40.0 \ \Omega) = 10.0 \ \text{V}$$

Finally, we can see that the current in the 120.0-Ω resistor is

$$I_{120} = \frac{V_{AB}}{120 \ \Omega} = \frac{10.0 \ \text{V}}{120 \ \Omega} = \frac{8.33 \times 10^{-2}}{\text{A}}$$

b. The power delivered to the 120.0-Ω resistor is given by Equation 20.6b as

$$P = I_{120}^2 R = \left(\frac{8.33 \times 10^{-2}}{\text{A}}\right)^2 (120.0 \ \Omega) = 0.833 \ \text{W}$$

63. **SSM** **REASONING** To find the current, we will use Ohm’s law, together with the proper equivalent resistance. The coffee maker and frying pan are in series, so their equivalent resistance is given by Equation 20.16 as $R_{\text{coffee}} + R_{\text{pan}}$. This total resistance is in parallel with the resistance of the bread maker, so the equivalent resistance of the parallel combination can be obtained from Equation 20.17 as $R_p^{-1} = (R_{\text{coffee}} + R_{\text{pan}})^{-1} + R_{\text{bread}}^{-1}$. 

**SOLUTION** Using Ohm’s law and the expression developed above for $R_p^{-1}$, we find

$$I = \frac{V}{R_p} = V \left( \frac{1}{R_{\text{coffee}} + R_{\text{pan}}} + \frac{1}{R_{\text{bread}}} \right) = (120 \text{ V}) \left( \frac{1}{14 \Omega + 16 \Omega} + \frac{1}{23 \Omega} \right) = 9.2 \text{ A}$$

64. **REASONING** We will approach this problem in parts. The resistors that are in series will be combined according to Equation 20.16, and the resistors that are in parallel will be combined according to Equation 20.17.

**SOLUTION** The 1.00 \( \Omega \), 2.00 \( \Omega \) and 3.00 \( \Omega \) resistors are in series with an equivalent resistance of $R_s = 6.00 \Omega$.

This equivalent resistor of 6.00 \( \Omega \) is in parallel with the 3.00-\( \Omega \) resistor, so

$$\frac{1}{R_p} = \frac{1}{6.00 \Omega} + \frac{1}{3.00 \Omega}$$

$R_p = 2.00 \Omega$

This new equivalent resistor of 2.00 \( \Omega \) is in series with the 6.00-\( \Omega \) resistor, so $R_s' = 8.00 \Omega$.

$R_s'$ is in parallel with the 4.00-\( \Omega \) resistor, so

$$\frac{1}{R_p'} = \frac{1}{8.00 \Omega} + \frac{1}{4.00 \Omega}$$

$R_p' = 2.67 \Omega$

Finally, $R_p'$ is in series with the 2.00-\( \Omega \), so the total equivalent resistance is $4.67 \Omega$. 

65. **REASONING** When two or more resistors are in series, the equivalent resistance is given by Equation 20.16: \( R_s = R_1 + R_2 + R_3 + \ldots \). Likewise, when resistors are in parallel, the expression to be solved to find the equivalent resistance is given by Equation 20.17: \( \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \). We will successively apply these to the individual resistors in the figure in the text beginning with the resistors on the right side of the figure.

**SOLUTION** Since the 4.0-Ω and the 6.0-Ω resistors are in series, the equivalent resistance of the combination of those two resistors is 10.0 Ω. The 9.0-Ω and 8.0-Ω resistors are in parallel; their equivalent resistance is 4.24 Ω. The equivalent resistances of the parallel combination (9.0 Ω and 8.0 Ω) and the series combination (4.0 Ω and the 6.0 Ω) are in parallel; therefore, their equivalent resistance is 2.98 Ω. The 2.98-Ω combination is in series with the 3.0-Ω resistor, so that equivalent resistance is 5.98 Ω. Finally, the 5.98-Ω combination and the 20.0-Ω resistor are in parallel, so the equivalent resistance between the points A and B is 4.6 Ω.

66. **REASONING** Between points a and b there is only one resistor, so the equivalent resistance is \( R_{ab} = R \). Between points b and c the two resistors are in parallel. The equivalent resistance can be found from Equation 20.17:

\[
\frac{1}{R_{bc}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \quad \text{so} \quad R_{bc} = \frac{1}{2} R
\]

The equivalent resistance between a and b is in series with the equivalent resistance between b and c, so the equivalent resistance between a and c is

\[
R_{ac} = R_{ab} + R_{bc} = R + \frac{1}{2} R = \frac{3}{2} R
\]

Thus, we see that \( R_{ac} > R_{ab} > R_{bc} \).

**SOLUTION** Since the resistance is \( R = 10.0 \Omega \), the equivalent resistances are:

\[
R_{ab} = R = 10.0 \Omega
\]

\[
R_{bc} = \frac{1}{2} R = 5.00 \Omega
\]

\[
R_{ac} = \frac{3}{2} R = 15.0 \Omega
\]

67. **REASONING** The two resistors \( R_1 \) and \( R_2 \) are wired in series, so we can determine their equivalent resistance \( R_{12} \). The resistor \( R_3 \) is wired in parallel with the equivalent resistance \( R_{12} \), so the equivalent resistance \( R_{123} \) can be found. Finally, the resistor \( R_4 \) is wired in series with the equivalent resistance \( R_{123} \). With these observations, we can evaluate the equivalent resistance between the points A and B.
SOLUTION Since \( R_1 \) and \( R_2 \) are wired in series, the equivalent resistance \( R_{12} \) is

\[
R_{12} = R_1 + R_2 = 16 \Omega + 8 \Omega = 24 \Omega
\]  

(20.16)

The resistor \( R_3 \) is wired in parallel with the equivalent resistor \( R_{12} \), so the equivalent resistance \( R_{123} \) of this combination is

\[
\frac{1}{R_{123}} = \frac{1}{R_3} + \frac{1}{R_{12}} = \frac{1}{48 \Omega} + \frac{1}{24 \Omega} \quad \text{or} \quad R_{123} = 16 \Omega
\]  

(20.17)

The resistance \( R_4 \) is in series with the equivalent resistance \( R_{123} \), so the equivalent resistance \( R_{AB} \) between the points \( A \) and \( B \) is

\[
R_{AB} = R_4 + R_{123} = 26 \Omega + 16 \Omega = 42 \Omega
\]

68. REASONING The total power \( P \) delivered by the battery is related to the equivalent resistance \( R_{eq} \) connected between the battery terminals and to the battery voltage \( V \) according to Equation 20.6c: \( P = V^2 / R_{eq} \). We note that the combination of resistors in circuit A is also present in circuits B and C (see the shaded part of these circuits in the following drawings). In circuit B an additional resistor is in parallel with the combination from A. The equivalent resistance of resistances in parallel is always less than any of the individual resistances alone. Therefore, the equivalent resistance of circuit B is less than that of A. In circuit C an additional resistor is in series with the combination from A. The equivalent resistance of resistances in series is always greater than any of the individual resistances alone. Therefore, the equivalent resistance of circuit C is greater than that of A. We conclude then that the equivalent resistances are ranked C, A, B, with C the greatest and B the smallest.
Since the total power delivered by the battery is \( P = V^2 / R_{eq} \), it is inversely proportional to the equivalent resistance. The battery voltage \( V \) is the same in all three cases, so the power ranking is the reverse of the ranking deduced previously for \( R_{eq} \). In other words, we expect that, from greatest to smallest, the total power delivered by the battery is B, A, C.

**SOLUTION** The total power delivered by the battery is \( P = V^2 / R_{eq} \). The voltage is given, but we must determine the equivalent resistance in each case. In circuit A each branch of the parallel combination consists of two resistances \( R \) in series. Thus, the resistance of each branch is \( R_{eq} = R + R = 2R \), according to Equation 20.16. The two parallel branches have an equivalent resistance that can be determined from Equation 20.17 as

\[
\frac{1}{R_A} = \frac{1}{2R} + \frac{1}{2R} \quad \text{or} \quad R_A = R
\]

In circuit B the resistance of circuit A is in parallel with an additional resistance \( R \). According to Equation 20.17, the equivalent resistance of this combination is

\[
\frac{1}{R_B} = \frac{1}{R_A} + \frac{1}{R} = \frac{1}{R} + \frac{1}{R} \quad \text{or} \quad R_B = \frac{1}{2} R
\]

In circuit C the resistance of circuit A is in series with an additional resistance \( R \). According to Equation 20.16, the equivalent resistance of this combination is

\[
R_C = R_A + R = R + R = 2R
\]

We can now use \( P = V^2 / R_{eq} \) to find the total power delivered by the battery in each case:

**[Circuit A]**

\[
P = \frac{V^2}{R} = \frac{(6.0 \text{ V})^2}{9.0 \text{ }\Omega} = 4.0 \text{ W}
\]

**[Circuit B]**

\[
P = \frac{V^2}{\frac{1}{2} R} = \frac{(6.0 \text{ V})^2}{\frac{1}{2}(9.0 \text{ }\Omega)} = 8.0 \text{ W}
\]

**[Circuit C]**

\[
P = \frac{V^2}{2R} = \frac{(6.0 \text{ V})^2}{2(9.0 \text{ }\Omega)} = 2.0 \text{ W}
\]

These results are as expected.
69. **REASONING** When two or more resistors are in series, the equivalent resistance is given by Equation 20.16: \( R_s = R_1 + R_2 + R_3 + \ldots \). When resistors are in parallel, the expression to be solved to find the equivalent resistance is Equation 20.17: \( \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \).

We will use these relations to determine the eight different values of resistance that can be obtained by connecting together the three resistors: 1.00, 2.00, and 3.00 Ω.

**SOLUTION** When all the resistors are connected in series, the equivalent resistance is the sum of all three resistors and the equivalent resistance is 6.00 Ω. When all three are in parallel, we have from Equation 20.17, the equivalent resistance is 0.545 Ω.

We can also connect two of the resistors in parallel and connect the parallel combination in series with the third resistor. When the 1.00 and 2.00-Ω resistors are connected in parallel and the 3.00-Ω resistor is connected in series with the parallel combination, the equivalent resistance is 3.67 Ω. When the 1.00 and 3.00-Ω resistors are connected in parallel and the 2.00-Ω resistor is connected in series with the parallel combination, the equivalent resistance is 2.75 Ω. When the 2.00 and 3.00-Ω resistors are connected in parallel and the 1.00-Ω resistor is connected in series with the parallel combination, the equivalent resistance is 2.20 Ω.

We can also connect two of the resistors in series and put the third resistor in parallel with the series combination. When the 1.00 and 2.00-Ω resistors are connected in series and the 3.00-Ω resistor is connected in parallel with the series combination, the equivalent resistance is 1.50 Ω. When the 1.00 and 3.00-Ω resistors are connected in series and the 2.00-Ω resistor is connected in parallel with the series combination, the equivalent resistance is 1.33 Ω. Finally, when the 2.00 and 3.00-Ω resistors are connected in series and the 1.00-Ω resistor is connected in parallel with the series combination, the equivalent resistance is 0.833 Ω.

---

70. **REASONING** The power \( P \) dissipated in each resistance \( R \) is given by Equation 20.6b as \( P = I^2R \), where \( I \) is the current. This means that we need to determine the current in each resistor in order to calculate the power. The current in \( R_1 \) is the same as the current in the equivalent resistance for the circuit. Since \( R_2 \) and \( R_3 \) are in parallel and equal, the current in \( R_1 \) splits into two equal parts at the junction A in the circuit.

**SOLUTION** To determine the equivalent resistance of the circuit, we note that the parallel combination of \( R_2 \) and \( R_3 \) is in series with \( R_1 \). The equivalent resistance of the parallel combination can be obtained from Equation 20.17 as follows:
\frac{1}{R_p} = \frac{1}{576 \Omega} + \frac{1}{576 \Omega} \quad \text{or} \quad R_p = 288 \Omega

This 288-\Omega resistance is in series with \( R_1 \), so that the equivalent resistance of the circuit is given by Equation 20.16 as

\[ R_{\text{eq}} = 576 \Omega + 288 \Omega = 864 \Omega \]

To find the current from the battery we use Ohm’s law:

\[ I = \frac{V}{R_{\text{eq}}} = \frac{120.0 \text{ V}}{864 \Omega} = 0.139 \text{ A} \]

Since this is the current in \( R_1 \), Equation 20.6b gives the power dissipated in \( R_1 \) as

\[ P_1 = I_1^2 R_1 = (0.139 \text{ A})^2 (576 \Omega) = 11.1 \text{ W} \]

\( R_2 \) and \( R_3 \) are in parallel and equal, so that the current in \( R_1 \) splits into two equal parts at the junction A. As a result, there is a current of \( \frac{1}{2} \) (0.139 A) in \( R_2 \) and in \( R_3 \). Again using Equation 20.6b, we find that the power dissipated in each of these two resistors is

\[ P_2 = I_2^2 R_2 = \left( \frac{1}{2} \right)^2 (0.139 \text{ A})^2 (576 \Omega) = 2.78 \text{ W} \]

\[ P_3 = I_3^2 R_3 = \left( \frac{1}{2} \right)^2 (0.139 \text{ A})^2 (576 \Omega) = 2.78 \text{ W} \]

71. **SSM REASONING** The power \( P \) delivered to the circuit is, according to Equation 20.6c, \( P = \frac{V^2}{R_{12345}} \), where \( V \) is the voltage of the battery and \( R_{12345} \) is the equivalent resistance of the five-resistor circuit. The voltage and power are known, so that the equivalent resistance can be calculated. We will use our knowledge of resistors wired in series and parallel to evaluate \( R_{12345} \) in terms of the resistance \( R \) of each resistor. In this manner we will find the value for \( R \).

**SOLUTION** First we note that all the resistors are equal, so \( R_1 = R_2 = R_3 = R_4 = R_5 = R \). We can find the equivalent resistance \( R_{12345} \) as follows. The resistors \( R_3 \) and \( R_4 \) are in series, so the equivalent resistance \( R_{34} \) of these two is \( R_{34} = R_3 + R_4 = 2R \). The resistors \( R_2 \), \( R_{34} \), and \( R_5 \) are in parallel, and the reciprocal of the equivalent resistance \( R_{2345} \) is

\[ \frac{1}{R_{2345}} = \frac{1}{R_2} + \frac{1}{R_{34}} + \frac{1}{R_5} = \frac{1}{2R} + \frac{1}{R} = \frac{5}{2R} \]

so \( R_{2345} = \frac{2R}{5} \). The resistor \( R_1 \) is in series with \( R_{2345} \), and the equivalent resistance of this combination is the equivalent resistance of the circuit. Thus, we have

\[ R_{12345} = R_1 + R_{2345} = R + \frac{2R}{5} = \frac{7R}{5} \]
The power delivered to the circuit is

\[ P = \frac{V^2}{R_{12345}} = \frac{V^2}{\left(\frac{7R}{5}\right)} \]

Solving for the resistance \( R \), we find that

\[ R = \frac{5V^2}{7P} = \frac{5(45 \text{ V})^2}{7(58 \text{ W})} = 25 \Omega \]

72. **REASONING** The resistance \( R \) of each of the identical resistors determines the equivalent resistance \( R_{eq} \) of the entire circuit, both in its initial form (six resistors) and in its final form (five resistors). We will calculate the initial and final equivalent circuit resistances by replacing groups of resistors that are connected either in series or parallel. The resistances of the replacement resistors are found either from Equation 20.16 for resistors connected in series, or from Equation 20.17, for resistors connected in parallel.

Once the equivalent resistance \( R_{eq} \) of the circuit is determined, the current \( I \) supplied by the battery (voltage \( V \)) is found from \( I = \frac{V}{R_{eq}} \) (Equation 20.2). We are given the decrease \( \Delta I \) in the battery current, which is equal to the final current \( I_f \) minus the initial current \( I_0 \):

\[ \Delta I = I_f - I_0 = -1.9 \text{ A} \]

The algebraic sign of \( \Delta I \) is negative because the final current is smaller than the initial current.

**SOLUTION** Beginning with the circuit in its initial form, we see that resistors 1, 3, and 5 are connected in series (see the drawing). These three resistors, according to Equation 20.16, may be replaced with a single resistor \( R_S \) that has three times the resistance \( R \) of a single resistor:

\[ R_S = 3R \]

The resistor \( R_S \) is connected in parallel with the resistor \( R_4 \) (see the drawing), so these two resistors may be replaced by an equivalent resistor \( R_p \) found from Equation 20.17:
After making this replacement, the three remaining resistors \( (R_2, R_3, \text{and } R_6) \) are connected in series across the battery (see the drawing). The initial equivalent resistance \( R_{eq,0} \) of the entire circuit, then, is found from Equation 20.16:

\[
R_{eq,0} = R_p + R_2 + R_4 = \frac{3R}{4} + 2R = \frac{11R}{4}
\]

Next, we consider the circuit after the resistor \( R_4 \) has been removed. The remaining five resistors are connected in series (see the drawing). From Equation 20.16, then, the final equivalent resistance \( R_{eq,f} \) of the entire circuit is five times the resistance \( R \) of a single resistor:

\[
R_{eq,f} = 5R
\]

From \( I = \frac{V}{R_{eq}} \) (Equation 20.2), the initial and final battery currents are

\[
I_0 = \frac{V}{R_{eq,0}} \quad \text{and} \quad I_f = \frac{V}{R_{eq,f}}
\]

Substituting Equations (2), (3), and (4) into Equation (1), we obtain

\[
\Delta I = I_f - I_0 = \frac{V}{R_{eq,f}} - \frac{V}{R_{eq,0}} = \frac{V}{5R} - \frac{V}{\frac{11R}{4}} = \frac{V}{R} \left( \frac{1}{5} - \frac{4}{11} \right) = -\frac{9V}{55R}
\]

Solving Equation (5) for \( R \) yields

\[
R = \frac{-9V}{55\Delta I} = -\frac{9(35\text{ V})}{55(-1.9\text{ A})} = 3.0\ \Omega
\]

73. **SSM Reasoning** The terminal voltage of the battery is given by \( V_{\text{terminal}} = \text{Emf} - Ir \), where \( r \) is the internal resistance of the battery. Since the terminal voltage is observed to be one-half of the emf of the battery, we have \( V_{\text{terminal}} = \frac{\text{Emf}}{2} \) and \( I = \frac{\text{Emf}}{2r} \). From Ohm's law, the equivalent resistance of the circuit is \( R = \frac{\text{Emf}}{I} = 2r \). We can also find the equivalent resistance of the circuit by considering that the identical bulbs are in parallel across the battery terminals, so that the equivalent resistance of the \( N \) bulbs is found from

\[
\frac{1}{R_p} = \frac{N}{R_{\text{bulb}}} \quad \text{or} \quad R_p = \frac{R_{\text{bulb}}}{N}
\]
This equivalent resistance is in series with the battery, so we find that the equivalent resistance of the circuit is

\[
R = 2r = \frac{R_{\text{bulb}}}{N} + r
\]

This expression can be solved for \( N \).

**SOLUTION** Solving the above expression for \( N \), we have

\[
N = \frac{R_{\text{bulb}}}{2r - r} = \frac{R_{\text{bulb}}}{r} = \frac{15 \Omega}{0.50 \Omega} = 30
\]

74. **REASONING** The following drawing shows the battery (emf = \( \xi \)), its internal resistance \( r \), and the 1.40-\( \Omega \) resistor. The voltage between the terminals of the battery is the voltage \( V_{AB} \) between the points \( A \) and \( B \) in the drawing. This voltage is not equal to the 9.00-V emf of the battery, because part of the emf is needed to make the current \( I \) go through the internal resistance. Ohm’s law states that this part of the emf is \( Ir \). The 1.40-\( \Omega \) resistor and the internal resistance are in series, so the current can be determined from \( \xi = I(r + R) \), which is Ohm’s law as applied to the entire series circuit.

\[
R = 1.40 \Omega
\]

\[
I
\]

\[
A \quad - \quad r \quad + \quad + \quad - \\
V_{\text{AB}} = 9.00 \text{ V} \quad 8.30 \text{ V}
\]

**SOLUTION** The terminal voltage \( V_{AB} \) is equal to the emf \( \xi \) of the battery minus the voltage across the internal resistance \( r \), which is \( Ir \) (Equation 20.2): \( V_{AB} = \xi - Ir \). Solving this equation for \( r \) gives

\[
r = \frac{\xi - V_{AB}}{I}
\]

(1)

To determine the current \( I \), we resort to Ohm’s law as applied to the entire series circuit, which is \( \xi = I(r + R) \). Solving this expression for \( I \) and substituting the result into Equation (1) reveals that

\[
r = \frac{\xi - V_{AB}}{I} = \frac{\xi - V_{AB}}{\xi / (r + R)} = \frac{(\xi - V_{AB})(r + R)}{\xi}
\]

Solving this result for \( r \), we find that

\[
r = \left( \frac{\xi - V_{AB}}{V_{AB}} \right) R = \left[ \frac{(9.00 \text{ V}) - (8.30 \text{ V})}{8.30 \text{ V}} \right] (1.40 \Omega) = 0.12 \Omega
\]
75. **REASONING** The following drawing shows the battery (emf = $\xi$), its internal resistance $r$, and the light bulb (represented as a resistor). The voltage between the terminals of the battery is the voltage $V_{AB}$ between the points $A$ and $B$ in the drawing. This voltage is not equal to the emf of the battery, because part of the emf is needed to make the current $I$ go through the internal resistance. Ohm’s law states that this part of the emf is $Ir$. The current can be determined from the relation $P = IV_{AB}$, since the power $P$ delivered to the light bulb and the voltage $V_{AB}$ across it are known.

![Diagram of electrical circuit with battery, resistor, and light bulb]

**SOLUTION** The terminal voltage $V_{AB}$ is equal to the emf $\xi$ of the battery minus the voltage across the internal resistance $r$, which is $Ir$ (Equation 20.2): $V_{AB} = \xi - Ir$. Solving this equation for the emf gives

$$\xi = V_{AB} + Ir$$

The current also goes through the light bulb, and it is related to the power $P$ delivered to the bulb and the voltage $V_{AB}$ according to $I = P/V_{AB}$ (Equation 20.6a). Substituting this expression for the current into Equation (1) gives

$$\xi = V_{AB} + Ir = V_{AB} + \left(\frac{P}{V_{AB}}\right)r$$

$$= 11.8 \text{ V} + \left(\frac{24 \text{ W}}{11.8 \text{ V}}\right)(0.10 \Omega) = 12.0 \text{ V}$$

76. **REASONING** When the terminal voltage of the battery (emf = 9.00 V) is 8.90 V, the voltage drop across the internal resistance $r$ is $9.00 \text{ V} - 8.90 \text{ V} = 0.10 \text{ V}$. According to Ohm’s law, this voltage drop is the current $I$ times the internal resistance. Thus, Ohm’s law will allow us to calculate the current.

**SOLUTION** Using Ohm’s law we find that

$$I = \frac{V}{r} = \frac{0.10 \text{ V}}{0.012 \Omega} = 8.3 \text{ A}$$
77. **REASONING** The voltage \( V \) across the terminals of a battery is equal to the emf of the battery minus the voltage \( V_r \) across the internal resistance of the battery: \( V = \text{Emf} - V_r \). Therefore, we have that

\[
\text{Emf} = V + V_r \tag{1}
\]

The power \( P \) being dissipated by the internal resistance is equal to the product of the voltage \( V_r \) across the internal resistance and the current \( I \): \( P = IV_r \) (Equation 20.6a). Therefore, we can express the voltage \( V_r \) across the internal resistance as

\[
V_r = \frac{P}{I} \tag{2}
\]

**SOLUTION** Substituting Equation (2) into Equation (1), we obtain

\[
\text{Emf} = V + V_r = V + \frac{P}{I} = 23.4 \text{ V} + \frac{34.0 \text{ W}}{55.0 \text{ A}} = 24.0 \text{ V}
\]

78. **REASONING** The power \( P \) delivered to a resistor such as the light bulb is found from

\[
P = I^2R \tag{20.6b}
\]

where \( I \) is the current in the resistor and \( R \) is the resistance. When either battery is connected to the bulb, the circuit can be modeled as a single-loop circuit with the battery emf in series with two resistors, namely, the internal resistance \( r \) and the resistance \( R \) of the bulb. As the two resistors are in series, their equivalent resistance \( R_S \) is given by

\[
R_S = r + R \tag{Equation 20.16}
\]

The current \( I \) supplied by a battery connected to a resistor \( R_S \) is, according to Ohm’s law,

\[
I = \frac{V}{R_S} \tag{20.2}
\]

where \( V \) is equal to the emf of the battery. Thus, \( V \) is the same for both circuits, since both batteries have the same emf, even though their internal resistances (\( r_{\text{wet}}, r_{\text{dry}} \)) are different.

**SOLUTION** Applying Equation 20.6b separately to each circuit, we obtain

\[
P_{\text{wet}} = I_{\text{wet}}^2R \quad \text{and} \quad P_{\text{dry}} = I_{\text{dry}}^2R \tag{2}
\]

Taking the ratio of Equations (2) eliminates the bulb resistance \( R \), yielding

\[
\frac{P_{\text{wet}}}{P_{\text{dry}}} = \frac{I_{\text{wet}}^2}{I_{\text{dry}}^2} \Rightarrow \frac{I_{\text{wet}}^2}{I_{\text{dry}}^2} = \frac{P_{\text{wet}}}{P_{\text{dry}}} \tag{3}
\]

Substituting Equation (20.2) for both \( I_{\text{wet}} \) and \( I_{\text{dry}} \) into Equation (3), we obtain
Replacing the equivalent series resistances in Equation (4) by Equation (1), we find that

\[
\frac{P_{\text{wet}}}{P_{\text{dry}}} = \left( \frac{\frac{V}{I}}{\frac{V}{I}} \right)^2 = \left( \frac{R_{\text{S,dry}}}{R_{\text{S,wet}}} \right)^2
\]

(4)

Solving for the current gives \[ I = 0.38 \text{ A} \].

b. The voltage between points A and B is

\[ V_{AB} = 30.0 \text{ V} - (0.38 \text{ A})(27 \text{ \Omega}) = 2.0 \times 10^1 \text{ V} \]

c. Point B is at the higher potential.

80. **Reasoning** Because all currents in the diagram flow from left to right (see the drawing), currents \( I_1 \) and \( I_2 \) both flow into junction A. Therefore, the current I in resistor R, which flows out of junction A, is, by Kirchhoff’s junction rule, equal to the sum of the other two currents:

\[ I = I_1 + I_2 \]  \hspace{1cm} (1)

The current \( I_1 \) is given. To determine \( I_2 \), we note that the resistors \( R_1 \) and \( R_2 \) are connected in parallel, and therefore must have the same potential difference \( V_{12} = V_1 = V_2 \) across them.
We will use Ohm’s law \( V = IR \) (Equation 20.2) to determine the potential difference \( V_{12} \) and then the current \( I_2 \).

**SOLUTION** Using \( V = IR \) (Equation 20.2), we express the voltage \( V_{12} \) across the resistors \( R_1 \) and \( R_2 \) in terms of currents and resistances as

\[
V_{12} = I_1 R_1 = I_2 R_2 \quad (2)
\]

Solving Equation (2) for \( I_2 \) yields

\[
I_2 = \frac{I_1 R_1}{R_2} \quad (3)
\]

Substituting Equation (3) into Equation (1), we obtain

\[
I = I_1 + I_2 = I_1 + \frac{I_1 R_1}{R_2} = I_1 \left(1 + \frac{R_1}{R_2}\right) = (3.00 \text{ A}) \left(1 + \frac{2.70 \Omega}{4.40 \Omega}\right) = 4.84 \text{ A}
\]

**81. REASONING AND SOLUTION** Label the currents with the resistor values. Take \( I_3 \) to the right, \( I_2 \) to the left and \( I_1 \) to the right. Applying the loop rule to the top loop (suppressing the units) gives

\[
I_1 + 2.0 I_2 = 1.0 \quad (1)
\]

and to the bottom loop gives

\[
2.0 I_2 + 3.0 I_3 = 5.0 \quad (2)
\]

Applying the junction rule to the left junction gives

\[
I_2 = I_1 + I_3 \quad (3)
\]

Solving Equations (1), (2) and (3) simultaneously, we find \( I_2 = 0.73 \text{ A} \).

The positive sign shows that the assumed direction is correct. That is, to the **left**.

**82. REASONING** In part \( a \) of the drawing in the text, the current goes from left-to-right through the resistor. Since the current always goes from a higher to a lower potential, the left end of the resistor is + and the right end is –. In part \( b \), the current goes from right-to-left through the resistor. The right end of the resistor is + and the left end is –. The potential drops and rises for the two cases are:

<table>
<thead>
<tr>
<th></th>
<th>Potential drops</th>
<th>Potential rises</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part ( a )</td>
<td>( IR )</td>
<td>( V )</td>
</tr>
<tr>
<td>Part ( b )</td>
<td>( V )</td>
<td>( IR )</td>
</tr>
</tbody>
</table>
**SOLUTION** Since the current \( I \) goes from left-to-right through the 3.0-\( \Omega \) and 4.0-\( \Omega \) resistors, the left end of each resistor is + and the right end is −. The current goes through the 5.0-\( \Omega \) resistor from right-to-left, so the right end is + and the left end is −. Starting at the upper left corner of the circuit, and proceeding clockwise around it, Kirchhoff’s loop rule is written as

\[
\left(3.0 \, \Omega\right)I + 12 \, V + \left(4.0 \, \Omega\right)I + \left(5.0 \, \Omega\right)I = 36 \, V
\]

Potential drops
Potential rises

Solving this equation for the current gives \( I = 2.0 \, A \)

83. **REASONING** This problem can be solved by using Kirchhoff’s loop rule. We begin by drawing a current through each resistor. The drawing shows the directions chosen for the currents. The directions are arbitrary, and if any one of them is incorrect, then the analysis will show that the corresponding value for the current is negative.

We mark the two ends of each resistor with plus and minus signs that serve as an aid in identifying the potential drops and rises for the loop rule, recalling that conventional current is always directed from a higher potential (+) toward a lower potential (−). Thus, given the directions chosen for \( I_1 \) and \( I_2 \), the plus and minus signs must be those shown in the drawing. We then apply Kirchhoff’s loop rule to the top loop (ABCF) and to the bottom loop (FCDE) to determine values for the currents \( I_1 \) and \( I_2 \).

**SOLUTION** Applying Kirchhoff’s loop rule to the top loop (ABCF) gives

\[
\frac{V_1 + I_2R_2}{I_1R_1} = I_1R_1
\]

Similarly, for the bottom loop (FCDE),

\[
\frac{V_2}{I_2} = I_2R_2
\]
Solving Equation (2) for \( I_2 \) gives

\[
I_2 = \frac{V_2}{R_2} = \frac{12 \text{ V}}{2.0 \Omega} = 6.0 \text{ A}
\]

Since \( I_2 \) is a positive number, the current in the resistor \( R_2 \) goes from left to right, as shown in the drawing. Solving Equation (1) for \( I_1 \) and substituting \( I_2 = \frac{V_2}{R_2} \) into the resulting expression yields

\[
I_1 = \frac{V_1 + I_2 R_2}{R_1} = \frac{V_1 + \left( \frac{V_2}{R_2} \right) R_2}{R_1} = \frac{V_1 + V_2}{R_1} = \frac{4.0 \text{ V} + 12 \text{ V}}{8.0 \Omega} = 2.0 \text{ A}
\]

Since \( I_1 \) is a positive number, the current in the resistor \( R_1 \) goes from left to right, as shown in the drawing.

---

**REASONING** We begin by labeling the currents in the three resistors. The drawing below shows the directions chosen for these currents. The directions are arbitrary, and if any of them is incorrect, then the analysis will show that the corresponding value for the current is negative.

We then mark the resistors with the plus and minus signs that serve as an aid in identifying the potential drops and rises for the loop rule, recalling that conventional current is always directed from a higher potential (+) toward a lower potential (−). Thus, given the directions chosen for \( I_1, I_2, \) and \( I_3 \), the plus and minus signs must be those shown in the drawing. We can now use Kirchhoff’s rules to find the voltage across the 5.0-Ω resistor.

**SOLUTION** Applying the loop rule to the left loop (and suppressing units for convenience) gives

\[
5.0 I_1 + 10.0 I_3 + 2.0 = 10.0
\]
Similarly, for the right loop,

$$10.0I_2 + 10.0I_3 + 2.0 = 15.0$$  \hfill (2)

If we apply the junction rule to the upper junction, we obtain

$$I_1 + I_2 = I_3$$  \hfill (3)

Subtracting Equation (2) from Equation (1) gives

$$5.0I_1 - 10.0I_2 = -5.0$$  \hfill (4)

We now multiply Equation (3) by 10 and add the result to Equation (2); the result is

$$10.0I_1 + 20.0I_2 = 13.0$$  \hfill (5)

If we then multiply Equation (4) by 2 and add the result to Equation (5), we obtain

$$20.0I_1 = 3.0$$, or solving for $I_1$, we obtain $I_1 = 0.15$ A. The fact that $I_1$ is positive means that the current in the drawing has the correct direction. The voltage across the 5.0-$\Omega$ resistor can be found from Ohm's law:

$$V = (0.15 \text{ A})(5.0 \Omega) = 0.75 \text{ V}$$

Current flows from the higher potential to the lower potential, and the current through the 5.0-$\Omega$ flows from left to right, so the left end of the resistor is at the higher potential.

---

85. **REASONING** In preparation for applying Kirchhoff’s rules, we now choose the currents in each resistor. The directions of the currents are arbitrary, and should they be incorrect, the currents will turn out to be negative quantities. Having chosen the currents, we also mark the ends of the resistors with the plus and minus signs that indicate that the currents are directed from higher (+) toward lower (−) potential. These plus and minus signs will guide us when we apply Kirchhoff’s loop rule.

**SOLUTION** Applying the junction rule to junction B, we find
Chapter 20  Problems

1. Applying the loop rule to loop ABCD (going clockwise around the loop), we obtain
\[
\frac{I_1}{\text{Into junction}} + \frac{2.00 \, \Omega}{\text{Potential drops}} = \frac{I_2}{\text{Out of junction}} + \frac{4.00 \, \Omega}{\text{Potential rises}} + 3.00 \, V
\]

(1)

2. Applying the loop rule to loop BEFC (going clockwise around the loop), we obtain
\[
\frac{I_2}{\text{Into junction}} + \frac{8.00 \, \Omega}{\text{Potential drops}} + \frac{9.00 \, V}{\text{Potential rises}} + \frac{4.00 \, \Omega}{\text{Potential rises}} = \frac{I_3}{\text{Out of junction}} + \frac{6.00 \, V}{\text{Potential rises}} = 0
\]

(2)

3. Substituting \( I_2 \) from Equation (1) into Equation (3) gives
\[
\left( \frac{I_1 + I_3}{2.00 \, \Omega} \right) + 9.00 \, V + \frac{I_3}{4.00 \, \Omega} + 6.00 \, V = 0
\]

(4)

4. Solving Equation (2) for \( I_1 \) gives
\[
I_1 = 4.50 \, A + I_3 (2.00)
\]

This result may be substituted into Equation (4) to show that
\[
\left[ 4.50 \, A + I_3 (2.00) \right] (8.00 \, \Omega) + I_3 (12.00 \, \Omega) + 15.00 \, V = 0
\]

\[
I_3 (28.00 \, \Omega) + 51.00 \, V = 0 \quad \text{or} \quad I_3 = \frac{-51.00 \, V}{28.00 \, \Omega} = -1.82 \, A
\]

The minus sign indicates that the current in the 4.00-\( \Omega \) resistor is directed downward, rather than upward as selected arbitrarily in the drawing.

86. REASONING
a. Currents \( I_5 \) and \( I_2 \) both go into junction \( C \) (see the drawing). This combined current \( I_5 + I_2 \) passes through the battery and splits up again at junction \( A \), with \( I_1 = 9.0 \, A \) going to the resistor \( R_1 \), and \( I_3 = 12.0 \, A \) going to the resistor \( R_3 \). By Kirchoff’s junction rule, therefore, the sum of the first pair of currents must equal the sum of the second pair of currents.
b. We will apply Kirchhoff’s loop rule to loop $CAB$ (see the drawing) and solve for the resistance $R_2$, which is the only unknown variable that appears in the loop rule. The + and − signs on either end of the resistors indicate that current flows from higher potential (+) to lower potential (−). They do not indicate that the ends of the resistors are charged in any way.

c. We will find $R_3$ by writing out Kirchhoff’s loop rule for the loop $CAED$ (see the drawing).

**SOLUTION**

a. As noted in the **REASONING**, the sum of the currents flowing into the battery must equal the sum of the currents flowing out of the battery:

$$I_2 + I_5 = I_1 + I_3$$

so

$$I_5 = I_1 + I_3 - I_2 = 9.0 \, \text{A} + 12.0 \, \text{A} - 6.0 \, \text{A} = 15.0 \, \text{A}$$

b. To apply Kirchhoff’s loop rule to loop $CAB$ (see the drawing), we imagine traversing the loop counter-clockwise. Observing the + and − signs encountered along the way, we see that the potential rises when we cross the battery, and drops when we cross each resistor. The sum of the potential rises must equal the sum of the potential drops, so we have that

$$V = I_1 R_1 + I_2 R_2$$

(1)

Solving Equation (1) for $R_2$, we obtain

$$I_2 R_2 = V - I_1 R_1$$

or

$$R_2 = \frac{V - I_1 R_1}{I_2} = \frac{75.0 \, \text{V} - (9.0 \, \text{A})(4.0 \, \Omega)}{6.0 \, \text{A}} = 6.5 \, \Omega$$

c. This time, we apply the loop rule to loop $CAED$ (see the drawing), traversing it clockwise. This procedure yields

$$V = I_3 R_3 + I_5 R_5$$

(2)

Solving Equation (2) for $R_3$, we obtain

$$I_3 R_3 = V - I_5 R_5$$

or

$$R_3 = \frac{V - I_5 R_5}{I_3} = \frac{75.0 \, \text{V} - (15.0 \, \text{A})(2.2 \, \Omega)}{12.0 \, \text{A}} = 3.5 \, \Omega$$

---

87. **SSM** **REASONING** As discussed in Section 20.11, some of the current (6.20 mA) goes directly through the galvanometer and the remainder $I$ goes through the shunt resistor. Since the resistance of the coil $R_C$ and the shunt resistor $R$ are in parallel, the voltage across each is the same. We will use this fact to determine how much current goes through the shunt.
resistor. This value, plus the 6.20 mA that goes through the galvanometer, is the maximum current that this ammeter can read.

**SOLUTION** The voltage across the coil resistance is equal to the voltage across the shunt resistor, so

$$\left(6.20 \times 10^{-3} \text{ A}\right)\left(20.0 \Omega\right) = (I)\left(24.8 \times 10^{-3} \Omega\right)$$

Voltage across coil resistance Voltage across shunt resistor

So $I = 5.00 \text{ A}$. The maximum current is $5.00 \text{ A} + 6.20 \text{ mA} = 5.01 \text{ A}$.

88. **REASONING** The following drawing shows a galvanometer that is being used as a nondigital voltmeter to measure the potential difference between the points A and B. The coil resistance is $R_C$ and the series resistance is $R$. The voltage $V_{AB}$ between the points A and B is equal to the voltage across the coil resistance $R_C$ plus the voltage across the series resistance $R$.

![Galvanometer Diagram](image)

**SOLUTION** Ohm’s law (Equation 20.2) gives the voltage across the coil resistance as $IR_C$ and that across the series resistor as $IR$. Thus, the voltage between A and B is

$$V_{AB} = IR_C + IR$$

Solving this equation for $R$ yields

$$R = \frac{V_{AB} - IR_C}{I} = \frac{10.0 \text{ V} - (0.400 \times 10^{-3} \text{ A})(60.0 \Omega)}{0.400 \times 10^{-3} \text{ A}} = 2.49 \times 10^4 \Omega$$

89. **REASONING** According to Ohm’s law and under full-scale conditions, the voltage $V$ across the equivalent resistance $R_{eq}$ of the voltmeter is $V = IR_{eq}$, where $I$ is the full-scale current of the galvanometer. The full-scale current is not given in the problem statement. However, it is the same for both voltmeters. By applying Ohm’s law to each voltmeter, we will be able to eliminate the current algebraically and calculate the full-scale voltage of voltmeter B.

**SOLUTION** Applying Ohm’s law to each voltmeter under full-scale conditions, we obtain

$$V_A = IR_{eq,A} \quad \text{and} \quad V_B = IR_{eq,B}$$
Dividing these two equations and eliminating the full-scale current $I$ give

$$\frac{V_B}{V_A} = \frac{IR_{\text{eq},B}}{IR_{\text{eq},A}} = \frac{R_{\text{eq},B}}{R_{\text{eq},A}}$$

or

$$V_B = V_A \frac{R_{\text{eq},B}}{R_{\text{eq},A}} = (50.0 \, \text{V}) \frac{1.44 \times 10^5 \, \Omega}{2.40 \times 10^5 \, \Omega} = 30.0 \, \text{V}$$

90. **REASONING** The drawing at the right shows the galvanometer (G), the coil resistance $R_C$ and the shunt resistance $R$. The full-scale current $I_G$ through the galvanometer and the current $I_S$ in the shunt resistor are also shown. Note that $R_C$ and $R$ are in parallel, so that the voltage across each of them is the same. Our solution is based on this fact. The equivalent resistance of the ammeter is the parallel equivalent resistance of $R_C$ and $R$.

**SOLUTION** Expressing voltage as the product of current and resistance and noting that the voltages across $R_C$ and $R$ are the same, we have

$$\frac{I_G R_C}{I_S R} = \frac{V}{V}$$

Solving this equation for the full-scale current $I_G$ through the galvanometer gives

$$I_G = \frac{I_S R}{R_C}$$

In this result $I_S$ and $R_C$ are given, but the shunt resistance $R$ is unknown. However, the equivalent resistance is given as $R_p = 0.40 \, \Omega$, and it is the parallel equivalent resistance of $R_C$ and $R$:

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R_C}$$

(Equation 20.17). This equation can be solved for $R$ as follows:

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R_C} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_p} - \frac{1}{R_C} = \frac{R_C - R_p}{R_p R_C} \quad \text{or} \quad R = \frac{R_p R_C}{R_C - R_p}$$

Substituting this result for $R$ into the expression for $I_G$ gives

$$I_G = \frac{I_S R}{R_C} = \frac{I_S}{R_C} \left( \frac{R_p R_C}{R_C - R_p} \right) = I_S \left( \frac{R_p}{R_C - R_p} \right)$$

$$= \left(3.00 \times 10^{-3} \, \text{A} \right) \left[ \frac{0.40 \, \Omega}{(9.00 \, \Omega) - (0.40 \, \Omega)} \right] = 1.4 \times 10^{-4} \, \text{A}$$
91. **SSM REASONING AND SOLUTION** For the 20.0 V scale

\[ V_1 = I(R_1 + R_c) \]

For the 30.0 V scale

\[ V_2 = I(R_2 + R_c) \]

Subtracting and rearranging yields

\[ I = \frac{V_2 - V_1}{R_2 - R_1} = \frac{30.0 \text{ V} - 20.0 \text{ V}}{2930 \Omega - 1680 \Omega} = 8.00 \times 10^{-3} \text{ A} \]

Substituting this value into either of the equations for \( V_1 \) or \( V_2 \) gives \( R_c = 820 \Omega \).

92. **REASONING AND SOLUTION**

a. According to Ohm’s law (Equation 20.2, \( V = IR \)) the current in the circuit is

\[ I = \frac{V}{R + R} = \frac{V}{2R} \]

The voltage across either resistor is \( IR \), so that we find

\[ IR = \left( \frac{V}{2R} \right) R = \frac{V}{2} = \frac{60.0 \text{ V}}{2} = 30.0 \text{ V} \]

b. The voltmeter’s resistance is \( R_v = \frac{V}{I} = (60.0 \text{ V})/(5.00 \times 10^{-3} \text{ A}) = 12.0 \times 10^3 \Omega \), and this resistance is in parallel with the resistance \( R = 1550 \Omega \). The equivalent resistance of this parallel combination can be obtained as follows:

\[ \frac{1}{R_p} = \frac{1}{R_v} + \frac{1}{R} \quad \text{or} \quad R_p = \frac{R R_v}{R + R_v} = \frac{(1550 \Omega)(12.0 \times 10^3 \Omega)}{1550 \Omega + 12.0 \times 10^3 \Omega} = 1370 \Omega \]

The voltage registered by the voltmeter is \( IR_p \), where \( I \) is the current supplied by the battery to the series combination of the other 1550-\( \Omega \) resistor and \( R_p \). According to Ohm’s law

\[ I = \frac{60.0 \text{ V}}{1550\Omega + 1370\Omega} = 0.0205 \text{ A} \]

Thus, the voltage registered by the voltmeter is

\[ IR_p = (0.0205 \text{ A})(1370\Omega) = 28.1 \text{ V} \]

93. **REASONING** A capacitor with a capacitance \( C \) stores a charge \( q \) when connected between the terminals of a battery of voltage \( V \), according to \( q = CV \) (Equation 19.8). This equation can be used to calculate the voltage if \( q \) and \( C \) are known. In this problem we know that the two capacitors together store a total charge of \( 5.4 \times 10^{-5} \text{ C} \). This is also the charge stored by
the equivalent capacitor, which has the equivalent capacitance $C_p$ of the parallel combination, or $C_p = C_1 + C_2$ (Equation 20.18). Thus, we can determine the battery voltage by using Equation 19.8 with the given total charge and the equivalent capacitance obtained from Equation 20.18.

**SOLUTION** With $q = q_{\text{Total}}$ and $C = C_p = C_1 + C_2$ Equation 19.8 becomes

$$q_{\text{Total}} = C_p V = \left( C_1 + C_2 \right) V$$

or

$$V = \frac{q_{\text{Total}}}{C_1 + C_2} = \frac{5.4 \times 10^{-5} \text{ C}}{2.0 \times 10^{-6} \text{ F} + 4.0 \times 10^{-6} \text{ F}} = 9.0 \text{ V}$$

94. **REASONING** The capacitance $C$ of a parallel plate capacitor is $C = \kappa \varepsilon_0 A / d$ (Equation 19.10), where $\kappa$ is the dielectric constant of the material between the plates, $\varepsilon_0$ is the permittivity of free space (a constant), $A$ is the area of each plate, and $d$ is the separation between the plates. The equivalent capacitance $C_S$ of three capacitors in series can be determined according to

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

(Equation 20.19), where $C_1$, $C_2$, and $C_3$ are the individual capacitances. Applying Equation 19.10 to the equivalent capacitance, we recognize that the dielectric constant becomes $\kappa_S$, which is the quantity that we seek.

**SOLUTION** Using Equation 20.19 for the series combination and using Equation 19.10 for $C_S, C_1, C_2$, and $C_3$, we have

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \text{or} \quad \frac{1}{\kappa_S \varepsilon_0 A / d} = \frac{1}{\kappa_1 \varepsilon_0 A / d} + \frac{1}{\kappa_2 \varepsilon_0 A / d} + \frac{1}{\kappa_3 \varepsilon_0 A / d}$$

In this result $\kappa_S$ is the dielectric constant of the single equivalent capacitor. Note that the same combination of constants $\varepsilon_0 A / d$ appears in every term on each side of the equals sign and may be eliminated algebraically from the result. Thus, we obtain

$$\frac{1}{\kappa_S} = \frac{1}{\kappa_1} + \frac{1}{\kappa_2} + \frac{1}{\kappa_3} = \frac{1}{3.30} + \frac{1}{5.40} + \frac{1}{6.70} = 0.637 \quad \text{or} \quad \kappa_S = 1.57$$

95. **SSM REASONING** The equivalent capacitance $C_S$ of a set of three capacitors connected in series is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

(Equation 20.19). In this case, we know that the equivalent capacitance is $C_S = 3.00 \mu\text{F}$, and the capacitances of two of the individual capacitors in this series combination are $C_1 = 6.00 \mu\text{F}$ and $C_2 = 9.00 \mu\text{F}$. We will use Equation 20.19 to determine the remaining capacitance $C_3$.

**SOLUTION** Solving Equation 20.19 for $C_3$, we obtain
Therefore, the third capacitance is

\[ C_3 = \frac{1}{3.00 \, \mu\text{F} - 6.00 \, \mu\text{F} - 9.00 \, \mu\text{F}} = 18 \, \mu\text{F} \]

96. **REASONING** When capacitors are connected in parallel, each receives the entire voltage \( V \) of the battery. Thus, the total energy stored in the two capacitors is \( \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 \) (see Equation 19.11b). When the capacitors are connected in series, the sum of the voltages across each capacitor equals the battery voltage: \( V_1 + V_2 = V \). Thus, the voltage across each capacitor is series is less than the battery voltage, so the total energy, \( \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \), is less than when the capacitors are wired in parallel.

**SOLUTION**

a. The voltage across each capacitor is the battery voltage, or 60.0 V. The energy stored in both capacitors is

\[
\text{Total energy} = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 = \frac{1}{2} (C_1 + C_2) V^2
\]

\[
= \frac{1}{2} \left( 2.00 \times 10^{-6} \, \text{F} + 4.00 \times 10^{-6} \, \text{F} \right) \left( 60.0 \, \text{V} \right)^2 = 1.08 \times 10^{-2} \, \text{J}
\]

b. According to the discussion in Section 20.12, the total energy stored by capacitors in series is \( \text{Total energy} = \frac{1}{2} C_S V^2 \), where \( C_S \) is the equivalent capacitance of the series combination:

\[
\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.00 \times 10^{-6} \, \text{F}} + \frac{1}{4.00 \times 10^{-6} \, \text{F}}
\]

(20.19)

Solving this equation yields \( C_S = 1.33 \times 10^{-6} \, \text{F} \). The total energy is

\[
\text{Total energy} = \frac{1}{2} \left( 1.33 \times 10^{-6} \, \text{F} \right) \left( 60.0 \, \text{V} \right)^2 = 2.39 \times 10^{-3} \, \text{J}
\]

97. **REASONING** Our approach to this problem is to deal with the arrangement in parts. We will combine separately those parts that involve a series connection and those that involve a parallel connection.

**SOLUTION** The 24, 12, and 8.0-\( \mu\text{F} \) capacitors are in series. Using Equation 20.19, we can find the equivalent capacitance for the three capacitors:
\[
\frac{1}{C_s} = \frac{1}{24 \, \mu F} + \frac{1}{12 \, \mu F} + \frac{1}{8.0 \, \mu F}
\]
or
\[
C_s = 4.0 \, \mu F
\]

This 4.0-\(\mu F\) capacitance is in parallel with the 4.0-\(\mu F\) capacitance already shown in the text diagram. Using Equation 20.18, we find that the equivalent capacitance for the parallel group is

\[
C_p = 4.0 \, \mu F + 4.0 \, \mu F = 8.0 \, \mu F
\]

This 8.0-\(\mu F\) capacitance is between the 5.0 and the 6.0-\(\mu F\) capacitances and in series with them. Equation 20.19 can be used, then, to determine the equivalent capacitance between \(A\) and \(B\) in the text diagram:

\[
\frac{1}{C_s} = \frac{1}{5.0 \, \mu F} + \frac{1}{8.0 \, \mu F} + \frac{1}{6.0 \, \mu F}
\]
or
\[
C_s = \left[2.0 \, \mu F\right]
\]

98. **REASONING**

a. When capacitors are wired in parallel, the total charge \(q\) supplied to them is the sum of the charges supplied to the individual capacitors, or \(q = q_1 + q_2\). The individual charges can be obtained from \(q_1 = C_1 V\) and \(q_2 = C_2 V\), since the capacitances, \(C_1\) and \(C_2\), and the voltage \(V\) are known.

b. When capacitors are wired in series, the voltage \(V\) across them is equal to the sum of the voltages across the individual capacitors, or \(V = V_1 + V_2\). However, the charge \(q\) on each capacitor is the same. The individual voltages can be obtained from \(V_1 = q/C_1\) and \(V_2 = q/C_2\).

**SOLUTION**

a. Substituting \(q_1 = C_1 V\) and \(q_2 = C_2 V\) (Equation 19.8) into \(q = q_1 + q_2\), we have

\[
q = q_1 + q_2 = C_1 V + C_2 V = \left(C_1 + C_2\right) V
\]

\[
= \left(2.00 \times 10^{-6} \, F + 4.00 \times 10^{-6} \, F\right) \left(60.0 \, V\right) = 3.60 \times 10^{-4} \, C
\]

b. Substituting \(V_1 = q/C_1\) and \(V_2 = q/C_2\) (Equation 19.8) into \(V = V_1 + V_2\) gives

\[
V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2}
\]

Solving this relation for \(q\), we have

\[
q = \frac{V}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{60.0 \, V}{\frac{1}{2.00 \times 10^{-6} \, F} + \frac{1}{4.00 \times 10^{-6} \, F}} = \frac{8.00 \times 10^{-5} \, C}{\text{Answer}}
\]
99. **REASONING AND SOLUTION** The charges stored on capacitors in series are equal and equal to the charge separated by the battery. The total energy stored in the capacitors is

\[
\text{Energy} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2}
\]

\[
\text{Energy} = \frac{Q^2}{2} \left( \frac{1}{C_1} + \frac{1}{C_2} \right)
\]

According to Equation 20.19, the quantity in the parentheses is just the reciprocal of the equivalent capacitance \(C\) of the circuit, so

\[
\text{Energy} = \frac{Q^2}{2C}
\]

100. **REASONING AND SOLUTION** The 7.00 and 3.00-µF capacitors are in parallel. According to Equation 20.18, the equivalent capacitance of the two is \(7.00 \, \mu F + 3.00 \, \mu F = 10.0 \, \mu F\). This 10.0-µF capacitance is in series with the 5.00-µF capacitance. According to Equation 20.19, the equivalent capacitance of the complete arrangement can be obtained as follows:

\[
\frac{1}{C} = \frac{1}{10.0 \, \mu F} + \frac{1}{5.00 \, \mu F} = 0.300 \, (\mu F)^{-1} \quad \text{or} \quad C = \frac{1}{0.300 \, (\mu F)^{-1}} = 3.33 \, \mu F
\]

The battery separates an amount of charge

\[
Q = CV = (3.33 \times 10^{-6} \, F)(30.0 \, V) = 99.9 \times 10^{-6} \, C
\]

This amount of charge resides on the 5.00 µF capacitor, so its voltage is

\[
V_5 = (99.9 \times 10^{-6} \, C)/(5.00 \times 10^{-6} \, F) = 20.0 \, V
\]

The loop rule gives the voltage across the 3.00 µF capacitor to be

\[
V_3 = 30.0 \, V - 20.0 \, V = 10.0 \, V
\]

This is also the voltage across the 7.00 µF capacitor, since it is in parallel, so \(V_7 = 10.0 \, V\).

101. **SSM REASONING** When two or more capacitors are in series, the equivalent capacitance of the combination can be obtained from Equation 20.19,

\[
\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots
\]

Equation 20.18 gives the equivalent capacitance for two or more capacitors in parallel:

\[
C_p = C_1 + C_2 + C_3 + \ldots
\]

The energy stored in a capacitor is given by \(\frac{1}{2} CV^2\), according to Equation 19.11. Thus, the energy stored in the series combination is \(\frac{1}{2} C_s V_s^2\), where
\[
\frac{1}{C_s} = \frac{1}{7.0 \ \mu F} + \frac{1}{3.0 \ \mu F} = 0.476 \ (\mu F)^{-1} \quad \text{or} \quad C_s = \frac{1}{0.476 \ (\mu F)^{-1}} = 2.10 \ \mu F
\]

Similarly, the energy stored in the parallel combination is \( \frac{1}{2} C_p V_p^2 \) where
\[
C_p = 7.0 \ \mu F + 3.0 \ \mu F = 10.0 \ \mu F
\]

The voltage required to charge the parallel combination of the two capacitors to the same total energy as the series combination can be found by equating the two energy expressions and solving for \( V_p \).

**SOLUTION**  Equating the two expressions for the energy, we have
\[
\frac{1}{2} C_s V_s^2 = \frac{1}{2} C_p V_p^2
\]
Solving for \( V_p \), we obtain the result
\[
V_p = V_s \sqrt{\frac{C_s}{C_p}} = (24 \ V) \sqrt{\frac{2.10 \ \mu F}{10.0 \ \mu F}} = 11 \ V
\]

**102. REASONING AND SOLUTION**  Charge is conserved during the re-equilibrium. Therefore, using \( q_0 \) and \( q_f \) to denote the initial and final charges, respectively, we have
\[
q_{10} + q_{20} = 18.0 \ \mu C = q_{1f} + q_{2f} \quad (1)
\]
After equilibrium has been established the capacitors will have equal voltages across them, since they are connected in parallel. Thus, \( V_f = q_{1f}/C_1 = q_{2f}/C_2 \), which leads to
\[
q_{1f} = q_{2f}(C_1/C_2) = q_{2f} (2.00 \ \mu F)/(8.00 \ \mu F) = 0.250 \ q_{2f}
\]
Substituting this result into Equation (1) gives
\[
18.0 \ \mu C = 0.250 \ q_{2f} + q_{2f} \quad \text{or} \quad q_{2f} = 14.4 \ \mu C
\]
Hence,
\[
V_f = q_{2f}/C_2 = (14.4 \times 10^{-6} \ C)/(8.00 \times 10^{-6} \ F) = 1.80 \ V
\]

**103. SSM REASONING**  The charge \( q \) on a discharging capacitor in a \( RC \) circuit is given by Equation 20.22:  \( q = q_0 e^{-t/RC} \), where \( q_0 \) is the original charge at time \( t = 0 \ s \).  Once \( t \) (time for one pulse) and the ratio \( q/q_0 \) are known, this expression can be solved for \( C \).

**SOLUTION**  Since the pacemaker delivers 81 pulses per minute, the time for one pulse is
\[
\frac{1 \ min}{81 \ \text{pulses}} \times \frac{60.0 \ s}{1.00 \ min} = 0.74 \ s/\text{pulse}
\]
Since one pulse is delivered every time the fully-charged capacitor loses 63.2\% of its original charge, the charge remaining is 36.8\% of the original charge. Thus, we have \( q = (0.368)q_0 \), or \( q / q_0 = 0.368 \).

From Equation 20.22, we have
\[
\frac{q}{q_0} = e^{-t/RC}
\]
Taking the natural logarithm of both sides, we have,
\[
\ln \left( \frac{q}{q_0} \right) = -\frac{t}{RC}
\]
Solving for \( C \), we find
\[
C = \frac{-t}{R \ln(q/q_0)} = \frac{-(0.74 \text{ s})}{(1.8 \times 10^6 \ \Omega) \ln(0.368)} = 4.1 \times 10^{-7} \text{ F}
\]

104. **REASONING** The time constant \( \tau \) is related to the resistance \( R \) and capacitance \( C \), according to \( \tau = RC \) (Equation 20.21). The problem deals with two cases. In case A, we have a time constant \( \tau_A = 1.5 \text{ s} \), a resistance \( R_A = 2.0 \times 10^4 \ \Omega \), and a capacitance \( C \). In case B, we have a time constant \( \tau_B \), a resistance \( R_B = 5.2 \times 10^4 \ \Omega \), and a capacitance \( C \). We will apply Equation 20.21 to both cases and take advantage of the fact that the capacitance is the same in both.

**SOLUTION** Applying Equation 20.21 to both cases, we have
\[
\tau_A = R_A C \quad \text{and} \quad \tau_B = R_B C
\]
Dividing the equation for case B by the equation for case A gives
\[
\frac{\tau_B}{\tau_A} = \frac{R_B C}{R_A C} = \frac{R_B}{R_A}
\]
Note that the unknown capacitance \( C \) has been eliminated algebraically from this result. Solving for the unknown time constant \( \tau_B \) gives
\[
\tau_B = \tau_A \left( \frac{R_B}{R_A} \right) = (1.5 \text{ s}) \left( \frac{5.2 \times 10^4 \ \Omega}{2.0 \times 10^4 \ \Omega} \right) = 3.9 \text{ s}
\]

105. **REASONING** The time constant of an \( RC \) circuit is given by Equation 20.21 as \( \tau = RC \), where \( R \) is the resistance and \( C \) is the capacitance in the circuit. The two resistors are wired in parallel, so we can obtain the equivalent resistance by using Equation 20.17. The two capacitors are also wired in parallel, and their equivalent capacitance is given by
Equation 20.18. The time constant is the product of the equivalent resistance and equivalent capacitance.

**SOLUTION** The equivalent resistance of the two resistors in parallel is

\[
\frac{1}{R_p} = \frac{1}{2.0 \, \text{k}\Omega} + \frac{1}{4.0 \, \text{k}\Omega} = \frac{3}{4.0 \, \text{k}\Omega} \quad \text{or} \quad R_p = 1.3 \, \text{k}\Omega \tag{20.17}
\]

The equivalent capacitance is

\[
C_p = 3.0 \, \mu\text{F} + 6.0 \, \mu\text{F} = 9.0 \, \mu\text{F} \tag{20.18}
\]

The time constant for the charge to build up is

\[
\tau = R_p C_p = \left(1.3 \times 10^3 \, \Omega\right)\left(9.0 \times 10^{-6} \, \text{F}\right) = 1.2 \times 10^{-2} \, \text{s}
\]

106. **REASONING** The charging of a capacitor is described by Equation 20.20, which provides a direct solution to this problem.

**SOLUTION** According to Equation 20.20, in a series RC circuit the charge \(q\) on the capacitor at a time \(t\) is given by

\[
q = q_0 \left(1 - e^{-t/\tau}\right)
\]

where \(q_0\) is the equilibrium charge that has accumulated on the capacitor after a very long time and \(\tau\) is the time constant. For \(q = 0.800q_0\) this equation becomes

\[
q = 0.800q_0 = q_0 \left(1 - e^{-t/\tau}\right) \quad \text{or} \quad 0.200 = e^{-t/\tau}
\]

Taking the natural logarithm of both sides of this result gives

\[
\ln(0.200) = \ln\left(e^{-t/\tau}\right) \quad \text{or} \quad \ln(0.200) = -\frac{t}{\tau}
\]

Therefore, the number of time constants needed for the capacitor to be charged to 80.0% of its equilibrium charge is

\[
\frac{t}{\tau} = -\ln(0.200) = -(-1.61) = 1.61
\]

107. **REASONING** In either part of the drawing the time constant \(\tau\) of the circuit is \(\tau = RC_{\text{eq}}\), according to Equation 20.21, where \(R\) is the resistance and \(C_{\text{eq}}\) is the equivalent capacitance of the capacitor combination. We will apply this equation to both circuits. To obtain the equivalent capacitance, we will analyze the capacitor combination in parts. For the parallel capacitors \(C_p = C_1 + C_2 + C_3 + ...\) applies (Equation 20.18), while for the series capacitors \(C_\text{s}^{-1} = C_1^{-1} + C_2^{-1} + C_3^{-1} + ...\) applies (Equation 20.19).
**SOLUTION** Using Equation 20.21, we write the time constant of each circuit as follows:

\[ \tau_a = R C_{eq, a} \quad \text{and} \quad \tau_b = R C_{eq, b} \]

Dividing these two equations allows us to eliminate the unknown resistance algebraically:

\[ \frac{\tau_b}{\tau_a} = \frac{R C_{eq, b}}{R C_{eq, a}} \quad \text{or} \quad \frac{\tau_b}{\tau_a} = \tau_a \left( \frac{C_{eq, b}}{C_{eq, a}} \right) \]

(1)

To obtain the equivalent capacitance in part a of the drawing, we note that the two capacitors in series in each branch of the parallel combination have an equivalent capacitance \( C_S \) that can be determined using Equation 20.19

\[ \frac{1}{C_S} = \frac{1}{C} + \frac{1}{C} \quad \text{or} \quad C_S = \frac{1}{2} C \]

(2)

Using Equation 20.18, we find that the parallel combination in part a of the drawing has an equivalent capacitance of

\[ C_{eq, a} = \frac{1}{2} C + \frac{1}{2} C = C \]

(3)

To obtain the equivalent capacitance in part b of the drawing, we note that the two capacitors in series have an equivalent capacitance of \( \frac{1}{2} C \), according to Equation (2). The two capacitors in parallel have an equivalent capacitance of \( 2C \), according to Equation 20.18. Finally, then, we have a series combination of \( \frac{1}{2} C \) and \( 2C \), for which Equation 20.19 applies:

\[ \frac{1}{C_{eq, b}} = \frac{1}{\frac{1}{2} C} + \frac{1}{2C} = \frac{5}{2C} \quad \text{or} \quad C_{eq, b} = \frac{2}{5} C \]

(4)

Using Equations (3) and (4) in Equation (1), we find that

\[ \tau_b = \tau_a \left( \frac{C_{eq, b}}{C_{eq, a}} \right) = (0.72 \text{ s}) \left( \frac{2}{5} C \right) = 0.29 \text{ s} \]
108. **REASONING**

a. The power delivered to a resistor is given by Equation 20.6c as \( P = \frac{V^2}{R} \), where \( V \) is the voltage and \( R \) is the resistance. Because of the dependence of the power on \( V^2 \), doubling the voltage has a greater effect in increasing the power than halving the resistance. The following table shows the power for each circuit, given in terms of these variables, and confirms this fact. The table also gives the expected ranking, in decreasing order, of the power.

<table>
<thead>
<tr>
<th></th>
<th>Power</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>( P = \frac{V^2}{R} )</td>
<td>3</td>
</tr>
<tr>
<td>( b )</td>
<td>( P = \frac{V^2}{2R} )</td>
<td>4</td>
</tr>
<tr>
<td>( c )</td>
<td>( P = \frac{(2V)^2}{R} ) = ( \frac{4V^2}{R} )</td>
<td>1</td>
</tr>
<tr>
<td>( d )</td>
<td>( P = \frac{(2V)^2}{2R} = \frac{2V^2}{R} )</td>
<td>2</td>
</tr>
</tbody>
</table>

b. The current is given by Equation 20.2 as \( I = \frac{V}{R} \). Note that the current, unlike the power, depends linearly on the voltage. Therefore, either doubling the voltage or halving the resistance has the same effect on the current. The following table shows the current for the four circuits and confirms this fact. The table also gives the expected ranking, in decreasing order, of the current.

<table>
<thead>
<tr>
<th></th>
<th>Current</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>( I = \frac{V}{R} )</td>
<td>2</td>
</tr>
<tr>
<td>( b )</td>
<td>( I = \frac{V}{2R} )</td>
<td>3</td>
</tr>
<tr>
<td>( c )</td>
<td>( I = \frac{2V}{R} )</td>
<td>1</td>
</tr>
<tr>
<td>( d )</td>
<td>( I = \frac{2V}{2R} = \frac{V}{R} )</td>
<td>2</td>
</tr>
</tbody>
</table>
**SOLUTION**

a. Using the results from the *REASONING* and the values of $V = 12.0 \text{ V}$ and $R = 6.00 \text{ } \Omega$, we find that the power dissipated in each resistor is

<table>
<thead>
<tr>
<th></th>
<th>Power</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$P = \frac{V^2}{R} = \frac{(12.0 \text{ V})^2}{6.00 \text{ } \Omega} = 24.0 \text{ W}$</td>
<td>3</td>
</tr>
<tr>
<td>$b$</td>
<td>$P = \frac{V^2}{2R} = \frac{(12.0 \text{ V})^2}{2(6.00 \text{ } \Omega)} = 12.0 \text{ W}$</td>
<td>4</td>
</tr>
<tr>
<td>$c$</td>
<td>$P = \frac{4V^2}{R} = \frac{4(12.0 \text{ V})^2}{6.00 \text{ } \Omega} = 96.0 \text{ W}$</td>
<td>1</td>
</tr>
<tr>
<td>$d$</td>
<td>$P = \frac{2V^2}{R} = \frac{2(12.0 \text{ V})^2}{6.00 \text{ } \Omega} = 48.0 \text{ W}$</td>
<td>2</td>
</tr>
</tbody>
</table>

b. Using the results from part (b) and the values of $V = 12.0 \text{ V}$ and $R = 6.00 \text{ } \Omega$, we find that the current in each circuit is

<table>
<thead>
<tr>
<th></th>
<th>Current</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$I = \frac{V}{R} = \frac{12.0 \text{ V}}{6.00 \text{ } \Omega} = 2.00 \text{ A}$</td>
<td>2</td>
</tr>
<tr>
<td>$b$</td>
<td>$I = \frac{V}{2R} = \frac{12.0 \text{ V}}{2(6.00 \text{ } \Omega)} = 1.00 \text{ A}$</td>
<td>3</td>
</tr>
<tr>
<td>$c$</td>
<td>$I = \frac{2V}{R} = \frac{2(12.0 \text{ V})}{6.00 \text{ } \Omega} = 4.00 \text{ A}$</td>
<td>1</td>
</tr>
<tr>
<td>$d$</td>
<td>$I = \frac{2V}{2R} = \frac{2(12.0 \text{ V})}{2(6.00 \text{ } \Omega)} = 2.00 \text{ A}$</td>
<td>2</td>
</tr>
</tbody>
</table>

109. [SSM] **REASONING** To find the equivalent capacitance of the three capacitors, we must first, following $C_p = C_1 + C_2 + C_3 + \cdots$ (Equation 20.18), add the capacitances of the two parallel capacitors together. We must then combine the result of Equation 20.18 with the remaining capacitance in accordance with $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$ (Equation 20.19). As Equation 20.19 shows, combining capacitors in series decreases the overall capacitance, and the resulting equivalent capacitance $C_S$ is smaller than any of the capacitances being added.
Therefore, the way to maximize the overall equivalent capacitance is to choose the largest capacitance \( C_1 \) to be connected in series with the parallel combination \( C_{23} \) of the smaller capacitances.

**SOLUTION** When \( C_2 \) and \( C_3 \) are connected in parallel, their equivalent capacitance \( C_{23} \) is, from \( C_p = C_1 + C_2 + C_3 + \cdots \) (Equation 20.18),

\[
C_{23} = C_2 + C_3
\]

When the equivalent capacitance \( C_{23} \) is connected in series with \( C_1 \), the resulting equivalent capacitance \( C_S \) is, according to Equation 20.19 and Equation (1),

\[
C_S = \left( \frac{1}{C_S} \right)^{-1} = \left( \frac{1}{C_1} + \frac{1}{C_{23}} \right)^{-1} = \left( \frac{1}{C_1} + \frac{1}{C_2 + C_3} \right)^{-1} = \left( \frac{1}{67 \mu F} + \frac{1}{45 \mu F + 33 \mu F} \right)^{-1} = 36 \mu F
\]

110. **REASONING** Electric current is the amount of charge flowing per unit time (see Equation 20.1). Thus, the amount of charge is the current times the time. Furthermore, the potential difference is the difference in electric potential energy per unit charge (see Equation 19.4), so that, once the amount of charge has been calculated, we can determine the energy by multiplying the potential difference by the charge.

**SOLUTION**

a. According to Equation 20.1, the current \( I \) is the amount of charge \( \Delta q \) divided by the time \( \Delta t \), or \( I = \Delta q / \Delta t \). Therefore, the additional charge that passes through the machine in normal mode versus standby mode is

\[
q_{\text{additional}} = \left( \frac{I_{\text{normal}}}{\Delta q \text{ in normal mode}} \right) \Delta t - \left( \frac{I_{\text{standby}}}{\Delta q \text{ in standby mode}} \right) \Delta t = \left( \frac{I_{\text{normal}} - I_{\text{standby}}}{\Delta t} \right) \Delta t
\]

\[
= (0.110 \text{ A} - 0.067 \text{ A})(60.0 \text{ s}) = 2.6 \text{ C}
\]

b. According to Equation 19.4, the potential difference \( \Delta V \) is the difference \( \Delta \text{(EPE)} \) in the electric potential energy divided by the charge \( q_{\text{additional}} \) or \( \Delta V = \Delta \text{(EPE)}/q_{\text{additional}} \). As a result, the additional energy used in the normal mode as compared to the standby mode is

\[
\Delta \text{(EPE)} = q_{\text{additional}} \Delta V = (2.6 \text{ C})(120 \text{ V}) = 310 \text{ J}
\]

111. **SSM REASONING AND SOLUTION** Ohm’s law (Equation 20.2), \( V = IR \), gives the result directly:

\[
R = \frac{V}{I} = \frac{9.0 \text{ V}}{0.11 \text{ A}} = 82 \Omega
\]
112. **REASONING** First, we draw a current $I_1$ (directed to the right) in the 6.00-$\Omega$ resistor. We can express $I_1$ in terms of the other currents in the circuit, $I$ and 3.00 A, by applying the junction rule to the junction on the left; the sum of the currents into the junction must equal the sum of the currents out of the junction.

\[
\begin{align*}
  I_1 &= I_1 + 3.00 \text{ A} \\
  \text{Current into junction on left} &= \text{Current out of junction on left} \\
  I &= I_1 - 3.00 \text{ A}
\end{align*}
\]

In order to obtain values for $I$ and $V$ we apply the loop rule to the top and bottom loops of the circuit.

**SOLUTION** Applying the loop rule to the top loop (going clockwise around the loop), we have

\[
(3.00 \text{ A})(4.00 \Omega) + (3.00 \text{ A})(8.00 \Omega) = 24.0 \text{ V} + (I - 3.00 \text{ A})(6.00 \Omega)
\]

Potential drops

Potential rises

This equation can be solved directly for the current; $I = 5.00 \text{ A}$.

Applying the loop rule to the bottom loop (going counterclockwise around the loop), we have

\[
(I - 3.00 \text{ A})(6.00 \Omega) + 24.0 \text{ V} + I(2.00 \Omega) = V
\]

Potential drops

Potential rises

Substituting $I = 5.00 \text{ A}$ into this equation and solving for $V$ gives $V = 46.0 \text{ V}$.

113. **REASONING AND SOLUTION** From Equation 20.5 we have that $R = R_0[1 + \alpha(T - T_0)]$. Solving for $T$ gives

\[
T = T_0 + \frac{R}{R_0 - \alpha^{-1}} = 20.0 \degree C + \frac{99.6 \Omega}{125 \Omega - 1} = \boxed{-34.6 \degree C}
\]

114. **REASONING** The magnitude $q$ of the charge on one plate of a capacitor is given by Equation 19.8 as $q = CV_1$, where $C = 9.0 \mu\text{F}$ and $V_1$ is the voltage across the capacitor. Since the capacitor and the resistor $R_1$ are in parallel, the voltage across the capacitor is equal to the voltage across $R_1$. From Equation 20.2 we know that the voltage across the 4.0-$\Omega$ resistor is given by $V_1 = IR_1$, where $I$ is the current in the circuit. Thus, the charge can be expressed as

\[
q = CV_1 = C(I R_1)
\]
The current is equal to the battery voltage \( V \) divided by the equivalent resistance of the two resistors in series, so that

\[
I = \frac{V}{R_s} = \frac{V}{R_1 + R_2}
\]

Substituting this result for \( I \) into the equation for \( q \) yields

\[
q = C \left( \frac{V}{R_1 + R_2} \right) R_1 = C \left( \frac{V}{R_1 + R_2} \right) R_1
\]

**SOLUTION** The magnitude of the charge on one of the plates is

\[
q = C \left( \frac{V}{R_1 + R_2} \right) R_1 = \left( 9.0 \times 10^{-6} \text{ F} \right) \left( \frac{12 \text{ V}}{4.0 \Omega + 2.0 \Omega} \right) (4.0 \Omega) = 7.2 \times 10^{-5} \text{ C}
\]

115. **REASONING** Since only 0.100 mA out of the available 60.0 mA is needed to cause a full-scale deflection of the galvanometer, the shunt resistor must allow the excess current of 59.9 mA to detour around the meter coil, as the drawing at the right indicates. The value for the shunt resistance can be obtained by recognizing that the 50.0-Ω coil resistance and the shunt resistance are in parallel, both being connected between points \( A \) and \( B \) in the drawing. Thus, the voltage across each resistance is the same.

**SOLUTION** Expressing voltage as the product of current and resistance, we find that

\[
\frac{59.9 \times 10^{-3} \text{ A}}{\text{Voltage across shunt resistance}} \left( R \right) = \frac{0.100 \times 10^{-3} \text{ A}}{\text{Voltage across coil resistance}} \left( 50.0 \Omega \right)
\]

\[
R = \frac{\left( 0.100 \times 10^{-3} \text{ A} \right) (50.0 \Omega)}{59.9 \times 10^{-3} \text{ A}} = 0.0835 \Omega
\]

116. **REASONING** Ohm’s law relates the resistance \( R \) of either resistor to the current \( I \) in it and the voltage \( V \) across it:

\[
R = \frac{V}{I} \quad (20.2)
\]

Because the two resistors are in series, they must have the same current \( I \). We will, therefore, apply Equation 20.2 to the 86-Ω resistor to determine the current \( I \). Following that, we will use Equation 20.2 again, to obtain the potential difference across the 67-Ω resistor.
**SOLUTION**  Let \( R_1 = 86 \, \Omega \) be the resistance of the first resistor, which has a potential difference of \( V_1 = 27 \, V \) across it. The current \( I \) in this resistor, from Equation 20.2, is

\[
I = \frac{V_1}{R_1}
\]

(1)

Let \( R_2 = 67 \, \Omega \) be the resistance of the second resistor. Again employing Equation 20.2, the potential difference \( V_2 \) across this resistor is given by

\[
V_2 = IR_2
\]

(2)

Since the current in both resistors is the same, substituting Equation (1) into Equation (2) yields

\[
V_2 = IR_2 = \left( \frac{V_1}{R_1} \right) R_2 = \frac{27 \, V}{86 \, \Omega} \times 67 \, \Omega = 21 \, V
\]

117. **SSM REASONING**  Since we know that the current in the 8.00-\( \Omega \) resistor is 0.500 A, we can use Ohm's law \((V = IR)\) to find the voltage across the 8.00-\( \Omega \) resistor. The 8.00-\( \Omega \) resistor and the 16.0-\( \Omega \) resistor are in parallel; therefore, the voltages across them are equal. Thus, we can also use Ohm's law to find the current through the 16.0-\( \Omega \) resistor. The currents that flow through the 8.00-\( \Omega \) and the 16.0-\( \Omega \) resistors combine to give the total current that flows through the 20.0-\( \Omega \) resistor. Similar reasoning can be used to find the current through the 9.00-\( \Omega \) resistor.

**SOLUTION**

a. The voltage across the 8.00-\( \Omega \) resistor is \( V_8 = (0.500 \, A)(8.00 \, \Omega) = 4.00 \, V \). Since this is also the voltage that is across the 16.0-\( \Omega \) resistor, we find that the current through the 16.0-\( \Omega \) resistor is \( I_{16} = (4.00 \, V)/(16.0 \, \Omega) = 0.250 \, A \). Therefore, the total current that flows through the 20.0-\( \Omega \) resistor is

\[
I_{20} = 0.500 \, A + 0.250 \, A = 0.750 \, A
\]

b. The 8.00-\( \Omega \) and the 16.0-\( \Omega \) resistors are in parallel, so their equivalent resistance can be obtained from Equation 20.17, \( \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \), and is equal to 5.33 \( \Omega \). Therefore, the equivalent resistance of the upper branch of the circuit is \( R_{\text{upper}} = 5.33 \, \Omega + 20.0 \, \Omega = 25.3 \, \Omega \), since the 5.33-\( \Omega \) resistance is in series with the 20.0-\( \Omega \) resistance. Using Ohm's law, we find that the voltage across the upper branch must be \( V = (0.750 \, A)(25.3 \, \Omega) = 19.0 \, V \). Since the lower branch is in parallel with the upper branch, the voltage across both branches must be the same. Therefore, the current through the 9.00-\( \Omega \) resistor is, from Ohm's law,

\[
I_9 = \frac{V_{\text{lower}}}{R_9} = \frac{19.0 \, V}{9.00 \, \Omega} = 2.11 \, A
\]
118. **REASONING AND SOLUTION**

a. In the first case the parallel resistance of the 75.0 Ω and the 45.0 Ω resistors have an equivalent resistance that can be calculated using Equation 20.17:

\[
\frac{1}{R_p} = \frac{1}{75.0 \, \Omega} + \frac{1}{45.0 \, \Omega}
\]

or

\[
R_p = 28.1 \, \Omega
\]

Ohm’s law, Emf = \(IR\) gives

\[
\text{Emf} = (0.294 \, \text{A})(28.1 \, \Omega + r), \text{ or }
\]

\[
\text{Emf} = 8.26 \, \text{V} + (0.294 \, \text{A})r
\]

In the second case, Emf = (0.116 \, \text{A})(75.0 \, \Omega + r), or

\[
\text{Emf} = 8.70 \, \text{V} + (0.116 \, \text{A})r
\]

Multiplying Equation (1) by 0.116 \, \text{A}, Equation (2) by 0.294 \, \text{A}, and subtracting yields

\[
\text{Emf} = 8.99 \, \text{V}
\]

b. Substituting this result into Equation (1) and solving for \(r\) gives

\[
r = 2.5 \, \Omega
\]

119. **SSM REASONING** The resistance of one of the wires in the extension cord is given by Equation 20.3: \(R = \rho L / A\), where the resistivity of copper is \(\rho = 1.72 \times 10^{-8} \, \Omega \cdot \text{m}\), according to Table 20.1. Since the two wires in the cord are in series with each other, their total resistance is

\[
R_{\text{cord}} = R_{\text{wire 1}} + R_{\text{wire 2}} = 2\rho L / A.
\]

Once we find the equivalent resistance of the entire circuit (extension cord + trimmer), Ohm's law can be used to find the voltage applied to the trimmer.

**SOLUTION**

a. The resistance of the extension cord is

\[
R_{\text{cord}} = \frac{2\rho L}{A} = \frac{2(1.72 \times 10^{-8} \, \Omega \cdot \text{m})(46 \, \text{m})}{1.3 \times 10^{-6} \, \text{m}^2} = 1.2 \, \Omega
\]

b. The total resistance of the circuit (cord + trimmer) is, since the two are in series,

\[
R_s = 1.2 \, \Omega + 15.0 \, \Omega = 16.2 \, \Omega
\]

Therefore from Ohm's law (Equation 20.2: \(V = IR\)), the current in the circuit is

\[
I = \frac{V}{R_s} = \frac{120 \, \text{V}}{16.2 \, \Omega} = 7.4 \, \text{A}
\]

Finally, the voltage applied to the trimmer alone is (again using Ohm's law),
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**Chapter 20   Problems**

**120. REASONING** The equivalent resistance of the three devices in parallel is $R_p$, and we can find the value of $R_p$ by using our knowledge of the total power consumption of the circuit; the value of $R_p$ can be found from Equation 20.6c, $P = V^2 / R_p$. Ohm's law (Equation 20.2, $V = IR$) can then be used to find the current through the circuit.

**SOLUTION**

a. The total power used by the circuit is $P = 1650 \text{ W} + 1090 \text{ W} + 1250 \text{ W} = 3990 \text{ W}$. The equivalent resistance of the circuit is

$$R_p = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{3990 \text{ W}} = 3.6 \Omega$$

b. The total current through the circuit is

$$I = \frac{V}{R_p} = \frac{120 \text{ V}}{3.6 \Omega} = 33 \text{ A}$$

This current is larger than the rating of the circuit breaker; therefore, the breaker will open.

**121. SSM REASONING** The resistance of a metal wire of length $L$, cross-sectional area $A$ and resistivity $\rho$ is given by Equation 20.3: $R = \rho L / A$. The volume $V_2$ of the new wire will be the same as the original volume $V_1$ of the wire, where volume is the product of length and cross-sectional area. Thus, $V_1 = V_2$ or $A_1 L_1 = A_2 L_2$. Since the new wire is three times longer than the first wire, we can write

$$A_1 L_1 = A_2 L_2 = A_2 (3L_1) \quad \text{or} \quad A_2 = A_1 / 3$$

We can form the ratio of the resistances, use this expression for the area $A_2$, and find the new resistance.

**SOLUTION** The resistance of the new wire is determined as follows:

$$\frac{R_2}{R_1} = \frac{\rho L_2 / A_2}{\rho L_1 / A_1} = \frac{L_2 A_1}{L_1 A_2} = \frac{(3L_1) A_1}{L_1 (A_1 / 3)} = 9$$

Solving for $R_2$, we find that

$$R_2 = 9 R_1 = 9 (21.0 \Omega) = 189 \Omega$$
122. **REASONING** The length $L$ of the wire is related to its resistance $R$ and cross-sectional area $A$ by $L = AR/\rho$ (see Equation 20.3), where $\rho$ is the resistivity of tungsten. The resistivity is known (see Table 20.1), and the cross-sectional area can be determined since the radius of the wire is given. The resistance can be obtained from Ohm’s law as the voltage divided by the current.

**SOLUTION** The length $L$ of the wire is

$$L = \frac{AR}{\rho}$$

(1)

Since the cross-section of the wire is circular, its area is $A = \pi r^2$, where $r$ is the radius of the wire. According to Ohm’s law (Equation 20.2), the resistance $R$ is related to the voltage $V$ and current $I$ by $R = V/I$. Substituting the expressions for $A$ and $R$ into Equation (1) gives

$$L = \frac{AR}{\rho} = \frac{\pi r^2 \left(\frac{V}{I}\right)}{\rho} = \frac{\pi \left(0.0030 \times 10^{-3} \text{ m}\right)^2 \left(120 \text{ V} \right)}{\frac{1.24 \text{ A}}{5.6 \times 10^{-8} \Omega \cdot \text{m}}} = 0.049 \text{ m}$$

123. [SSM] **REASONING** The foil effectively converts the capacitor into two capacitors in series. Equation 19.10 gives the expression for the capacitance of a capacitor of plate area $A$ and plate separation $d$ (no dielectric): $C_0 = \varepsilon_0 A / d$. We can use this expression to determine the capacitance of the individual capacitors created by the presence of the foil. Then using the fact that the "two capacitors" are in series, we can use Equation 20.19 to find the equivalent capacitance of the system.

**SOLUTION** Since the foil is placed one-third of the way from one plate of the original capacitor to the other, we have $d_1 = (2/3)d$, and $d_2 = (1/3)d$. Then

$$C_1 = \frac{\varepsilon_0 A}{(2/3)d} = \frac{3\varepsilon_0 A}{2d}$$

and

$$C_2 = \frac{\varepsilon_0 A}{(1/3)d} = \frac{3\varepsilon_0 A}{d}$$

Since these two capacitors are effectively in series, it follows that

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{3\varepsilon_0 A/(2d)}{3\varepsilon_0 A/d} + \frac{3\varepsilon_0 A/d}{3\varepsilon_0 A} = \frac{3d}{3\varepsilon_0 A} = \frac{d}{\varepsilon_0 A}$$

But $C_0 = \varepsilon_0 A / d$, so that $d/(\varepsilon_0 A) = 1/C_0$, and we have
124. **REASONING AND SOLUTION** The mass \( m \) of the aluminum wire is equal to the density \( d \) of aluminum times the volume of the wire. The wire is cylindrical, so its volume is equal to the cross-sectional area \( A \) times the length \( L \); \( m = dAL \).

The cross-sectional area of the wire is related to its resistance \( R \) and length \( L \) by Equation 20.3; \( R = \rho L/A \), where \( \rho \) is the resistivity of aluminum (see Table 20.1). Therefore, the mass of the aluminum wire can be written as

\[
m = dAL = d \left( \frac{\rho L}{R} \right) L
\]

The resistance \( R \) is given by Ohm’s law as \( R = V/I \), so the mass of the wire becomes

\[
m = d \left( \frac{\rho L}{R} \right) L = \frac{d \rho L^2 I}{V}
\]

\[
m = \frac{(2700 \text{ kg/m}^3)(2.82 \times 10^{-8} \text{ } \Omega \cdot \text{m})(175 \text{ m})^2(125 \text{ A})}{0.300 \text{ V}} = 9.7 \times 10^2 \text{ kg}
\]