Measuring Your Latitude from the Angle of the Sun at Noon

Background: You can measure your latitude in earth's northern hemisphere by finding out the altitude of the celestial equator from the southern horizon, as seen in your view of the sky.

To find the angle to the celestial equator, you need to measure the angle of the sun in the sky at solar noon, and then add or subtract the declination of the sun. The declination is the angle between the sun and the celestial equator.

If this sounds confusing, you should review the celestial sphere concept in your book. Examine the celestial sphere diagram in the book that shows the celestial equator and celestial poles. Because the celestial equator is a projection of the earth's equator onto the celestial sphere, you can use it to measure your latitude on earth.

Outcomes: By performing this exercise, you will:

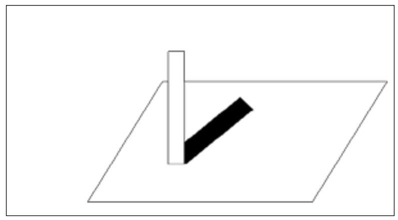
- Learn what solar noon ("true noon") really is
- Conduct an experiment in which you build and run the equipment yourself, demonstrating your ability to combine hands-on work with critical thinking
- Gain experience at making a trigonometric calculation on a calculator using the $\tan^{-1}$ function
- Confront the relationship between latitudes on earth and declinations on the celestial sphere
- Show how precisely you can measure your latitude on earth

Introduction: A gnomon is a vertical post that casts its shadow onto a flat surface. By measuring the exact length of the gnomon's shadow at noon, along with the exact length of the gnomon itself, you can determine your latitude on earth. To do so, you must also correct for the declination of the sun. The declination of the sun is its position relative to the celestial equator and can be looked up on a detailed star chart or a table of solar declination by date.

Observation and Measurement Procedure:

1. Construct a gnomon by using a straight stick, 12-inch ruler, or straight post or dowel. It should be at least 10 inches in length. Shorter gnomons lead to greater relative errors in measurement.
2. Mount the gnomon exactly vertically so that it stands straight up. The surface the gnomon casts its shadow on should be perfectly flat and horizontal, broad enough to encompass the shadow of the gnomon. You need to be able to clearly mark on the surface with a fine pencil.
3. Measure the exact length of the gnomon, the length it sticks up from the surface it is mounted on. Write the measurement down.

4. Starting at 11:30 Standard Time (which is in effect late fall, winter, and early spring -- it's up to you to know when), or 12:30 if Daylight Savings Time is in effect, mark the exact position of the tip of the gnomon's shadow. Measure its length and write it down, along with the time.
5. Keep marking and measuring the shadow length every five minutes for one full hour, starting before and continuing after the shortest shadow length for an hour total.
6. The most important length and time of the shadow is the length and time when the shadow of the gnomon reached its shortest length. That time is solar noon, true noon.



Calculation Procedure:

1. Divide the exact length of the gnomon by the exact length of its shadow at solar noon.
2. Find the angle that the sun was located above your southern horizon at solar noon. To find this angle, just take the inverse tangent of the ratio you calculated in step 1. The inverse tangent shows up on most calculators as $\tan^{-1}$. The inverse tangent, as you may recall, is the angle which has a tangent equal to the given number. Make sure your calculator is displaying and calculating angles in degrees, not in radians or other units.
3. Look up the declination of the sun on the day you made your measurement. The declination is the number of degrees north (+) or south (-) of the celestial equator. The declination of the sun can be found by using a detailed star chart, one that shows the ecliptic with dates on it, and also shows declination. Or you can find the declination of the sun by searching online for a table of solar declination for each day of the year. Make sure you get the sun's declination for the day you made your measurement.
4. Subtract the sun's declination from the angle you found in the previous step. The result is the altitude of the celestial equator above the south point on the horizon.
5. Subtract from 90° the previous angle (90° minus the angle from the horizon to the celestial equator). The result is your latitude on earth. Write it down. Turn it in, along with all your other observations. Make sure your name is on it before you turn it in.

The Photography Requirement: In addition, take a photograph of your gnomon instrument and turn it in, labeled and dated, printed on (or glued on) a piece of paper that is part of your submitted gnomon document.

Fill in the following table and essay questions (pages 3 and 4) and turn them in. Make sure your name is on each page you turn in.

Latitude on Earth Determined with a Gnomon and Solar Declination (see notes 1-5 after)	
Date:	City or nearest city:
[1]Exact length of gnomon:	
Exact Time:	Exact Length of Gnomon Shadow:
Exact Time:	Exact Length of Gnomon Shadow:
Exact Time:	Exact Length of Gnomon Shadow:
Exact Time:	Exact Length of Gnomon Shadow:
Exact Time:	Exact Length of Gnomon Shadow:
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Exact Time:	Exact Length of Gnomon Shadow:
Exact Time:	Exact Length of Gnomon Shadow:
Exact Time:	Exact Length of Gnomon Shadow:
Exact Time of Shortest Shadow (Time of Solar Noon):	
Exact Length of Shortest Shadow:	
[2]Ratio of (Gnomon Length)/(Shortest Shadow Length):	
[3]Angle of Sun Above Horizon: Tan^{-1} of Ratio:	
[4]Declination of Sun:	
[5]Your Latitude: 90° - [(Angle of Sun) - (Declination)] =	

Notes (see table above):

1. Measure in meters or a prefixed metric length unit, such as cm or mm. Do not measure in feet or inches.
    a. Be as precise as your measuring method allows. If you are using a ruler or measuring tape, that means you must visually estimate the last unit between the marked units on your ruler or tape. For example, if you use a ruler that has marks down to the mm, then you should estimate the lengths to the nearest tenth of a mm, unless you are not confident of such an estimate, in which case just measure to the nearest mm.

    b. A mm is the same thing as a tenth of a cm, by the way. For example, 25.4 cm = 254 mm. Either way of writing that length down, to the tenth of a cm or to the mm, would be acceptable.

    c. Remember that 25 does not equal 25.0, which itself does not equal 25.00.

       For example, if you are measuring to the nearest tenth (0.1) of a centimeter, don't write 25 cm if you have a length that looks to be exactly 25 centimeters; instead, write 25.0 cm.

2. If you write a decimal fraction such as 0.6588. be sure to put the zero before the decimal point. Do not write .6588, because that would not be considered correct.

3. Be sure your calculator is using degrees for its angle units. Write all angles in decimal degrees. Be sure to put the ° symbol after the number.

4. If you look up the declination of the Sun for your gnomon experiment date and find it listed in degrees, arc minutes, and possibly arc seconds as well, then convert the number of degrees entirely to decimal form. Divide the minutes by sixty and add the resulting decimal fraction to the whole number of degrees. Divide the minutes by 3600 and add that small decimal fraction to the preceding total. If the declination is negative (-), which means the Sun was south of the celestial equator, then the decimal fractions are also negative.

   CITE THE SOURCE, title and URL, from where you got your solar declination.

5. Your Latitude = 90° - [(Angle of Sun) - (Declination)].

   If the declination is negative (-), you will be subtracting a negative number from the Angle of Sun, which is the same as adding the declination angle as a positive (+) value to the Angle of Sun.

   For example, if the Angle of Sun is 33.2° and Declination is -10.1°, then

   90° - [(Angle of Sun) - (Declination)] = 90° - [33.2° – (-10.1°)] = 90° - (33.2° + 10.1°)

   = 90° - (43.3°) = <u>**N 46.7°**</u> YOU MUST SHOW YOUR WORK, LIKE IN THIS EXAMPLE.

   The N is because you need to say whether the latitude is north or south of the equator.

   Round off the latitude number to the same precision as your least precise length measurement.

Answer the following questions. Give complete explanations. Cite any sources.

1. Why must the entire shadow of the gnomon be cast onto a flat, horizontal surface?

2. Why is the shadow of the gnomon shortest at solar noon?

3. Why must the measurements of the length of the gnomon and the length of the shadow be made to as high a precision as possible?

4. Why, if the length measurements are made to a precision of XX.X cm, can the latitude not be listed to a greater precision than XX.X°?

5. If you were at the Tropic of Cancer on June 21, what would the length of your gnomon's shadow at solar noon have been? Explain.

Grading Rubric for Gnomon Assignment		
Attribute	Worth (% of total score)	Comment
Independent Performance	Up to 100%	Data not repeated in other student's assignments.
On time	Up to 100%	Assignments 1-2 days late may (or may not) earn a few points; later than that, zero points.
Length of experiment	Up to 50%	Measurements made for 1 full hour.
Precision	Up to 33%	Correctly records and handles decimals (and/or fractions).
Declination	Up to 20%	Able to figure out correct declination of Sun and include it in calculations.
Angle of Sun	Up to 20%	Calculated correct angle of Sun based on ratio.
Latitude	Up to 20%	Deduced accurate latitude.
Five Written Questions	Up to 40%	Each question answered adequately using complete sentences.
Labeled Photo	Up to 20%	Labeled or captioned, date-stamped photo of your gnomon instrument.